

# Mathematical Models Of Typical Elements Of Water Management Systems

Ilkhomjon Makhmudov<sup>1</sup>, Daniyar Jumamuratov<sup>2</sup>, Aybek Seytov<sup>3</sup>, Umidjon Sadiev<sup>4</sup>, Uktam Jovliev<sup>5</sup>, Jonibek Shonazarov<sup>6</sup>, Oybek Muminov<sup>7</sup>, Muzaffar Ruziev<sup>8</sup>, Muxtorbek Yusupov<sup>9</sup>

<sup>1</sup>Doctor of technical sciences, Professor, Scientific Research Institute of Irrigation and Water Problems, Uzbekistan.

<sup>2</sup>Candidate of technical sciences, Docent, Nukus branch of Navoi State Mining Institute,

<sup>3</sup>PhD, Docent, Scientific Research Institute of Irrigation and Water Problems, Uzbekistan.

<sup>4</sup>PhD, Scientific Research Institute of Irrigation and Water Problems, Uzbekistan.

<sup>5</sup>PhD, Scientific Research Institute of Irrigation and Water Problems, Uzbekistan.

<sup>6</sup>Assistant, National Research University "TIQXMMI" Counter irrigation and agricultural technologies Institute, Department "Melioration and melioration", Uzbekistan.

<sup>7</sup>Doctorate, Department of Civil engineering construction, faculty of Construction Fergana polytechnic institute, Uzbekistan.

<sup>8</sup>Scientific Research Institute of Irrigation and Water Problems, Uzbekistan.

<sup>9</sup>Chirchik State Pedagogical Institute, Uzbekistan.

**Abstract:** The article shows the results of developed mathematical models of typical elements of water management systems for the optimal management of water resources of large main canals with cascades of pumping stations and uses methods for system analysis of the process of water supply and water intake, modern methods for calculating the operating modes of pumping stations with long pipelines and calculating the operating modes of canal sections.

**Keywords:** mathematical model, unsteady flow of water, main canals, optimal control problems, fundamental solution, differential equations, hydrotechnical structure.

## Introduction

Uzbekistan is considered as one of the largest irrigation farming countries in Central Asia. Proper use of existing water and land resources can increase crop production and yields in the agriculture sector [1]. The problem of managing a water management facility differs in that it is necessary to solve the problems of collecting information about the management object from large territories and managing its links, which are located at long distances from each other (in some cases more than a hundred kilometers). This is especially typical during managing river basin facilities. This situation necessitates the use of systems for collecting, transmitting and processing information by objects located at large

distances, which consists of hardware and software.

## Materials and methods

Dynamic processes in the sections of the canals, limited by barrier structures are described by a one-dimensional system of Saint-Venant equations for the unsteady movement of water in the open canals. When using canals, it is important to assess quantitative indicators of the state of reliability associated with such adverse effects as wear of canal dams under the influence of dangerous filtration currents, subsidence, and elevation of canal sections relative to the area [2].

The characteristic form of the record for the unsteady movement of water in the sections of open canals has the following matrix form [3]:

$$S \frac{\partial U}{\partial t} + \Lambda S \frac{\partial U}{\partial x} = F(U, K, t) \quad (1)$$

where:

$$S = \begin{bmatrix} 1 & -B(v+c) \\ 1 & -B(v-c) \end{bmatrix}; \quad U = \begin{bmatrix} Q \\ z \end{bmatrix};$$

$$\Lambda = \begin{bmatrix} v-c & 0 \\ 0 & v+c \end{bmatrix}, \quad F = -BIv^2 - g\omega \frac{Q|Q|}{K}$$

Canal section - 1:

Boundary conditions at  $x=0$  of section - 1 [3]

$$Q_1(0, t) = F_0(t) = \mu_0 \omega_0(t) \sqrt{2g[z_{\text{upstream}}(t) - z_{\text{downstream}}(0, t)]} \quad (2)$$

where:  $\mu_0$  – discharge coefficient of the hydraulic structure located at the beginning of the canal section,  $\omega_0(t)$  – open hole area of shutter,  $z_{\text{upstream}}(t)$  – upstream water level mark,  $g$  – acceleration of gravity,  $z_{\text{downstream}}(0, t)$  – the mark of the water level at the beginning of the canal or the downstream of the hydraulic structure located at the beginning of the canal section.

Boundary conditions at  $x=l_1$  of section - 1 [4]

$$Q_1(l_1, t) = F_1(t) + F_2(t) + F_3(t),$$

$$F_1(t) = \mu_1 \omega_1(t) \sqrt{2g[z_1(l_1, t) - z_2(0, t)]},$$

$$F_2(t) = \mu_2 \omega_2(t) \sqrt{2g[z_1(l_1, t) - z_3(0, t)]},$$

$$F_3(t) = \mu_3 \omega_3(t) \sqrt{2g[z_1(l_1, t) - z_4(0, t)]},$$

(3)

where:  $\mu_i$  – discharge coefficient of the hydraulic structure located at the beginning of the canal section,  $\omega_i(t)$  – open hole area of shutter,  $z_i(0, t)$  – the mark of the water level at the beginning of the canal,  $z_i(l_i, t)$  – the mark of the water level at the end of the canal.

Structurally, the model of a hydraulic structure is represented by follow:

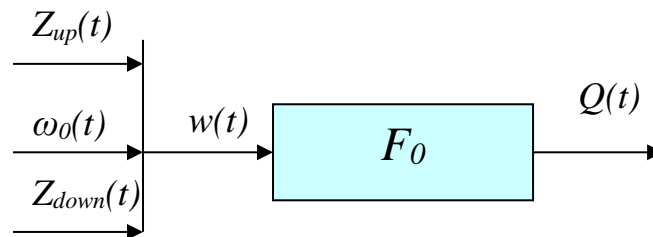


Fig.1. – Describing block of hydrotechnical structure

From the condition of conjugation of the canal sections, we use the boundary conditions for the canal sections. If the sections of the canal are interconnected by partitioning structures with a flooded outflow from under the shutter (Fig. – 2), the expressions for the boundary conditions have the following form:

As shown in [5,6], the processes taking place in the canal sections can be structurally represented as a block with distributed parameters (Fig. 2b).

where:  $w_{\text{chs}}(t) = \{Q_0(x), z_0(x), u_1(t), u_2(t), \text{ and } q(x, t)\}$ ,  $Q_{\text{chs}}(x, t) = \{Q(x, t), z(x, t)\}$  - output signals are the sequence of input signals of the block of the corresponding canal section.

$\mathcal{L}_A^{\text{chs}}$  – is an algorithmic operator of the canal section, uniquely connecting the sequence of input signals with the sequence of output signals. The algorithmic operator is a software module that determines the output signals from the given values of the input signals [7].



$$\Omega_T^i = \begin{cases} Q_j^i & J = \overline{1, K}; (i = \overline{1, N}), \quad N \leq M \\ H_T^j & J = \overline{1, K} \end{cases} \quad (5)$$

where:  $Q_j^i$  - argument of the pressure characteristic of the pipeline, i.e., upstream of the  $i$ -th pumping unit;  $K$  - number of points in the pressure characteristic;  $N$  - the number of operating pumping units;  $H_T^j = H + \nabla H_j$  - pressure characteristic function;  $\nabla H_j$  - pressure loss.

The exploitation characteristics of the pumping unit are presented as a family of curves depending on the height of the water at various angles of rotation of the impeller blades:

$$\Omega_e^i = \Omega_{H,Q,\phi} \cup \Omega_{H,\eta,\phi}, \quad i = \overline{1, N}, \quad (6)$$

where:

$$\Omega_{H,Q,\phi} = \begin{cases} Q_j^i & i = \overline{1, N}, (j = \overline{1, K}) \\ H_i & i = \overline{1, N}, \\ \phi_j & j = \overline{1, K}, \end{cases} \quad -$$

discharge characteristic of the pumping unit

$$\Omega_{H,\eta,\phi} = \begin{cases} \eta_j^i & i = \overline{1, N}, (j = \overline{1, K}) \\ H_i & i = \overline{1, N}, \\ \phi_j & j = \overline{1, K}, \end{cases} \quad - \text{ energy}$$

characteristic of the pumping unit;

$\phi_j$  - blade turning angle corresponding to the  $J$ -th curve;

$\eta_j^i$  - Efficiency of the 1st pumping unit for  $J$ -th curve.

Permissible area  $D$  of operation of the pumping unit in coordinates  $Q$ - $H$  is determined by the following external boundaries:

$$\left. \begin{aligned} D_{1 \max}^i &= \Omega_T^{i \max} \cap \Omega_{H,Q,\phi}^i, \\ D_{1 \min}^i &= \Omega_T^{i \min} \cap \Omega_{H,Q,\phi}^i, \\ D_{2 \max}^i &= \Omega_{H,Q,\phi \max}^i, \\ D_{2 \min}^i &= \Omega_{H,Q,\phi \min}^i, \end{aligned} \right\} \quad (7)$$

where:  $\Omega_T^{\max}$ ,  $\Omega_T^{\min}$  - pipeline characteristic at maximum and minimum geometric lifting height;  $\phi_{\max}$ ,  $\phi_{\min}$  - maximum and minimum angles of rotation of the blades of the pumping unit.

If the given values of  $Q$  and  $H$  are located inside the area  $D$ , then it is considered that the required water discharge can be provided by this unit, otherwise this mode cannot be implemented by this unit. When several units are operating, the boundaries of the allowable area are determined by summing the flow discharges within the boundaries of the areas at a constant lifting height.

Flow discharge, gauge height and efficiency operating pumping unit, i.e., the state of each operating pumping unit is characterized by a triple:

$$(z_{\text{upstream}}, z_{\text{downstream}}, \phi),$$

where:  $\phi_p^i$  - angle of rotation of the blades of the  $i$ -th operating pumping unit. Consequently, the flow discharge and efficiency of the  $i$ -th pumping unit is determined from the following expressions:

$$\begin{aligned} \Omega_p^i &= (\Omega_T^i \cap \Omega_{H,Q,\phi}) \cap \Omega_{H,Q,\eta}, \\ \phi_i &= \phi_i^p, \quad \Omega_{H,Q,\phi} \subset \Omega_3^i, \quad \Omega_{H,\eta,\phi}^i \subset \Omega_3^i \end{aligned} \quad (8)$$

The total discharge and power consumption for the pumping station as a whole is determined as the algebraic sum of the flow discharge and capacities of the operating unit [10]:

$$Q_{HC} = \sum_{i \in NP} Q_i, \quad N_{HC} = \sum_{i \in NP} N_i, \quad (9)$$

where:  $N_i = \gamma^H Q_i / 102 \eta_i$  /kW/-power of the  $i$ -th pumping unit;

$\gamma$  - volumetric weight of the pumped liquid.

Thus, the water flow and power consumption of the pumping station are determined by the following algorithmic dependencies:

$$\begin{aligned} Q_{ps}(\mathbf{t}) &= F_q(\mathbf{t}, N_i^p(\mathbf{t}), z_{\text{upstream}}(\mathbf{t}), z_{\text{downstream}}(\mathbf{t})) \\ N_{ps}(\mathbf{t}) &= F_n(\mathbf{t}, N_i^p(\mathbf{t}), z_{\text{upstream}}(\mathbf{t}), z_{\text{downstream}}(\mathbf{t})) \end{aligned} \quad (10)$$

where:  $N^p(\mathbf{t})$  - many operating pumping units,  $z_{\text{upstream}}(\mathbf{t})$  - upstream water level mark,  $z_{\text{downstream}}(\mathbf{t})$  - downstream water level mark.

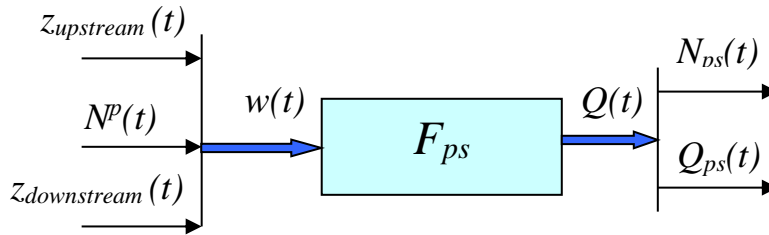


Fig.3. - Block describing the pumping station

The block describing the pumping station is shown in the figure here:  $w(t) = \{z_{upstream}(t), N^p(t)\}$  and  $z_{downstream}(t)$ ,  $Q(t) = \{N_{ps}(t), Q_{ps}(t)\}$  – sequence of input and output signals of the block of the corresponding reservoir.  $F_{ps}$  – algorithmic operator of the pumping station block [9,10,11]:

#### Mathematical model of the reservoir

The change in water volumes in the reservoir over time is described by the following differential equation [12,13]:

$$\frac{dW_i^R}{dt} = \sum_{j \in N_{inflow}} Q_j^{inflow} - \sum_{j \in N_{intake}} Q_j^{intake} - Q_i^L - Q_i^{discharge},$$

$$W_i^R(0) = W_0^R, \quad W^R = F_w(H^R), S^R = F_s(H^R), \quad t \in [0, T] \quad (11)$$

где  $W_i^R$  – reservoir water volume at a time  $t$ ;  $Q_j^{inflow}$  and  $Q_j^{intake}$  – water discharge rate of the  $j$ -th inflow and water intake from the reservoir;  $Q^L$  – intensity of water loss in the reservoir;  $Q^{discharge}$

– discharge of water from the reservoir;  $S^R$  – reservoir surface area,  $F_w(H^R)$  – volumetric characteristic of the reservoir,  $F_s(H^R)$  – areal characteristic of the reservoir.

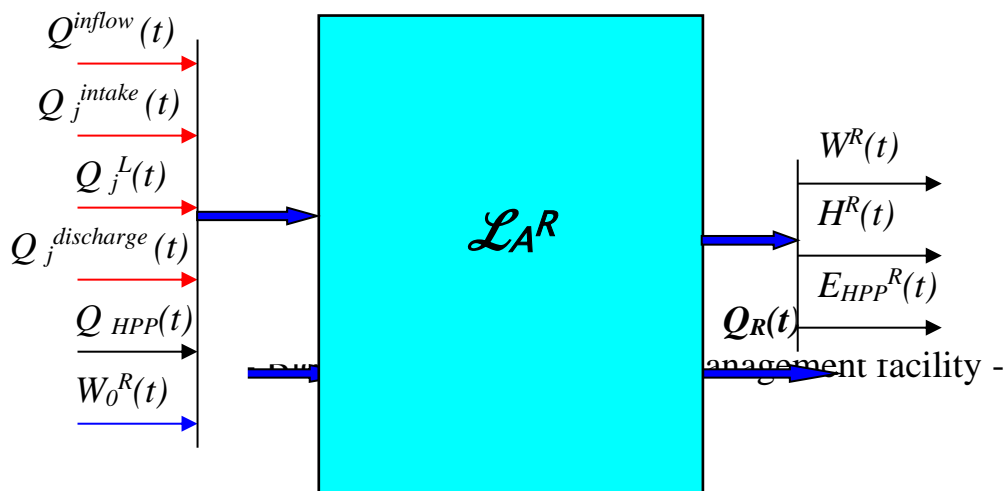
Many reservoirs are designed for complex purposes, which include hydroelectric power plants. For such reservoirs, descriptions of energy regimes are required in the model

The generation of electricity by a hydroelectric power plant is described by the following expression [14]:

$$E_{HPP}^R = 9,81\eta \int_0^t \Delta H^R(\tau) Q_{HPP}(\tau) d\tau \quad (12)$$

where:  $\Delta H(t)$  – pressure change over the time,  $Q_{HPP}(t)$  – water discharge flowing through the turbines of a hydroelectric power plant,  $\eta$  – HPP turbine efficiency.

In that way, by knowing  $Q_j^{inflow}(t)$ ,  $Q_j^{intake}(t)$ ,  $Q^L(t)$ ,  $Q^{discharge}(t)$ ,  $Q_{HPP}(t)$  and  $W_0^R$  – solving equation (1), and by expression (2) we can determine  $W^R(t)$ ,  $H^R(t)$  and  $E_{HPP}^R(t)$  and structurally present it as a block (Fig.4)



where:  $W_R(t) = \{Q_j^{inflow}(t), Q_j^{intake}(t), Q^L(t), Q^{discharge}(t), Q_{HPP}(t) \text{ and } W_0^R\}$ ,

$Q_R(t) = \{W^R(t), H^R(t) \text{ and } E_{HPP}^R(t)\}$  – the sequence of input and output signals of the block of the corresponding reservoir.

$\mathcal{L}_A^R$  - an algorithmic operator that uniquely links the sequence of input signals with the sequence of output signals.

### Mathematical model of a gauging station in the sections of the canal and rivers

The gauging station on sections of rivers and canals are built to unambiguously determine the water discharge according to the measured values of the water level on them. There is a special

calibration characteristic of the gauging station where there is a dependence in the form of a graph or table, determined on the basis of field measurements of water discharge at various level values.

Mathematically, this dependence is written as [15]:

$$Q = f(H), \quad (13)$$

где  $Q$  – water discharge,  $H$  water level.

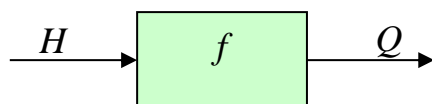


Fig.5. - Block describing the gauging station in the area of rivers and canals

### Results and Discussion

The main results of the article are:

- Mathematical models and algorithms have been developed for modeling processes in automatic water distribution control systems at typical water management facilities (hydropower structures, hydroelectric facilities and reservoirs)
- Recommendations have been developed for choosing a control scheme for typical water management facilities based on modeling processes in automatic control systems for typical water management facilities.

### Conclusion

In this article, typical elements of water management systems are selected, mathematical models, algorithms and software modules for modeling typical elements of water management systems for automation, collection and processing of data objects are developed.

The developed algorithms can be applied in the design and operation of automation systems, data collection and processing.

### References

1. Ilkhomjon Mahmudov, Umidjon Sadiev, Khurshid Lapasov, Azizbek Ernazarov, Shokhrukh Rustamov. Solution of the Filter Flow Problem by Analytical and Numerical Methods. Cite as: AIP Conference Proceedings 2432, 040006 (2022); <https://doi.org/10.1063/5.0090359>
2. Ilkhomjon Mahmudov, Umidjon Sadiev, Shokhrukh Rustamov. Basic Conditions for Determining the Hydraulic

Resistance to Friction in a Pipeline when a Mixture of Water and Suspended Sediments Moves. Cite as: AIP Conference Proceedings 2432, 040005 (2022); <https://doi.org/10.1063/5.0090349>

3. Ilkhomjon Ernazarovich Mahmudov, Aliev Mahmud Kuvatovich, Mahmudova Dildora Ernazarovna, Musayev Sharof Mamarajabovich, Rustamova Mukhlisa Muhtaraliyevna, Nematov Davlat Berdiyori o'g'li, Boboyorov Bekhruz Ixtiyor ugli. Development Of A High-Performance Technology For Mixing Ozone With Water For The Preparation Of Drinking Water From The Reservoir. Journal of Positive School Psychology 2022, Vol. 6, No. 5, 2921-2925 <http://journalppw.com>

4. Ilkhomjon Mahmudov, Akmal Mirzaev, Navruz Murodov, Azizbek Ernazarov, Adkham Rajabov, Musayev Sharof, Jasur Narziev, Bobur Ulug'bekov, Shokhrukh Ustemirov. Socio-Economic Situation In The Water Management Of The Republic Of Uzbekistan And The Regulatory-Legal And Economical Frameworks For The Implementing Of

Water-Saving Technologies. Journal of Positive School Psychology 2022, Vol. 6, No. 5, 2951-2955 <http://journalppw.com>

5. Ilkhomjon Mahmudov, Navruz Murodov, Azizbek Ernazarov, Uktam Jovliev, Musayev Sharof, Adkham Rajabov, Bobur Ulug'bekov, Shokhrukh Ustemirov. The Current State Of Irrigation Networks And Their Use In The Water Sector Of The Republic Of Uzbekistan. Journal of Positive School

Psychology 2022, Vol. 6, No. 5, 2947-2950  
<http://journalppw.com>

6. Ilkhomjon Ernazarovich Mahmudov, Mahmudova Dildora Ernazarovna, Aliev Mahmud Kuvatovich, Abdullaev Akhror Zhakhbarovich, Kamalova Saodat Nigmatjanovna, Musayev Sharof Mamarajabovich, Boboyorov Bekhruz Ixtiyor ug'li . Analysis Of Improved Methods For Determining Last Generations Of Pesticides In Water Water. Journal of Positive School Psychology 2022, Vol. 6, No. 5, 2926-2933  
<http://journalppw.com>

7. Ilkhomjon Mahmudov, Navruz Murodov, Akmal Mirzaev, Uktam Jovliev, Umidjon Sadiev, Musayev Sharof, Adkham Rajabov, Jasur Narziev, Muzaffar Ro'ziev. Probability-Statistical Model Of Reliability And Efficiency Of Irrigation Channels. Journal of Positive School Psychology 2022, Vol. 6, No. 5, 2956-2960  
<http://journalppw.com>

8. Ilkhomjon Ernazarovich .Mahmudov, Paluanov Daniyar, Umidjon Abdusamadovich Sadiev. TECHNICAL SOLUTIONS TO ENSURE THE SAFETY OF OPERATING HYDRAULIC ENGINEERING CONSTRUCTIONS. ASEAN Journal on Science & Technology for Development. Vol 39, No 4, 2022, 189-191 189  
[DOI 10.5281/zenodo.6583860](https://doi.org/10.5281/zenodo.6583860)

9. Ergash Kazakov, Uktam Jovliev, Gulomzhon Yakubov. Extension of tubular water discharge limitations with water flow extinguishers. INTERNATIONAL JOURNAL OF SCIENTIFIC & TECHNOLOGY RESEARCH VOLUME 8, ISSUE 12, DECEMBER 2019 ISSN 2277-8616.  
<https://www.ijstr.org/final-print/dec2019/Extension-Of-Tubular-Water-Discharge-Limitations-With-Water-Flow-Extinguishers.pdf>

10. Rakhimov, S., Seytov, A., Nazarov, B., Buvabekov, B., Optimal control of unstable water movement in canals of irrigation systems under conditions of discontinuity of water delivery to consumers. IOP Conf. Series: Materials Science and Engineering 883 (2020) 012065, Dagestan, 2020, IOP Publishing DOI:10.1088/1757-899X/883/1/012065 (№5, Scopus, IF=4,652)

11. Shavkat Rakhimov, Aybek Seytov, Nasiba Rakhimova, Bahrom Xonimqulov. Mathematical models of optimal distribution of water in main canals. 2020 IEEE 14th International Conference on Application of Information and Communication Technologies (AICT), INSPEC Accession Number: 20413548, IEEE Access, Tashkent, Uzbekistan, DOI:10.1109/AICT50176.2020.9368798 (AICT) pp. 1-4,(№ 5, Scopus, IF=3,557)

12. Rakhimov, S., Seytov, A., Sherbaev, M., Kudaybergenov, A., Khurramov, A. Algorithms for Solving the Problems of Optimizing Water Resources Management on a Reservoir Seasonal Regulation. AIP Conference Proceedings 2432, 060023 (2022);  
<https://doi.org/10.1063/5.0090412>  
 Published Online: 16 June 2022.

13. Rakhimov, S., Seytov, A., Kudaybergenov, A. Modeling and optimization of water supply processes at large pumping stations. Global and Stochastic Analysis, 2021, 8(3), стр. 57–62.

14. Seytov, A., Turayev, R., Jumamuratov, D., Kudaybergenov, A. Mathematical Models for Calculation of Limits in Water Resources Management in Irrigation Systems. International Conference on Information Science and Communications Technologies: Applications, Trends and Opportunities, ICISCT 2021, 2021

15. Narziev J.J., Maxmudov I.E., Paluanov D.T., Ernazarov A.I. Assessment of Probability Reliability of Hydro technical Structures during Operation Period., <http://openaccessjournals.eu/index.php/ijiaet/article/view/949>.