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Estimation Of The Stochastic Frontier Model Parameters With The Nonlinear Smooth Transfer Function

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Abstract

The purpose of the present study was to estimate the parameters of the new stochastic Frontier model by nonlinear smooth transfer function structure with compound error. In the stochastic Frontier model, the compound error consists of two statistical components: model error and technical inefficiency, so that the technical inefficiency of the function is assumed to be autocorrelated. For the parametric part, the nonlinear smooth transfer function was used using the Taylor series, and for the non-parametric part, the moderating factor was used to estimate the model parameters. The research model was evaluated using data on the number of patients admitted to hospitals for Covid 19 disease. The result showed that the parameter estimates are consistent and have less error than conventional models.

Keywords: Stochastic Estimation, Stochastic Frontier Model, Nonlinear Smooth Transfer Function, Functional Coefficient - Technical Inefficiency

Introduction

The classical time series prediction consists of estimating unknown parameters in a suitable model by presenting a model for the future. It should be noted that the nonlinear regression functions are the main element of nonlinear time series models, especially nonlinear autoregressive models. Therefore, in the nonlinear autoregressive model, the prediction of the unknown function f plays a key role. In this regard, researchers in the field of statistics and econometrics, using parametric and non-parametric methods, have been able to find an estimate for the function f(0). According to the time data conditions, although the use of parametric and non-parametric methods separately does not provide a suitable estimate for the f(0) function and has a relatively large mean squares of error (risk), the use of semi-parametric estimation methods, reduces the error of the predictor function. Therefore, the use of semi-parametric estimation in threshold first-order or higher-order functional autoregressive models contributes greatly to the good fit of nonlinear time series data (Gao, 2005).

The semi-parametric method is used to estimate the first-order or higher-order functional autoregressive model function for the better fit criterion on the functional time data. So, most of the previous articles are in the way of introducing ordinary autoregressive models and in particular functional autoregressive models (different linear and nonlinear functions) that have studied historically in such a way. Kargin and Onatsky (2008) proposed curve prediction by means of functional autoregressive models. They used the prediction of autoregressive processes for functional time data, using the technique of predictive factor resolving for the functions of the functional autoregressive model, and also tried to improve the estimation in such models.

Chen (2003) used the short-term functional autoregressive model for electricity price curve with functional estimation using conventional methods. Also, the nonparametric estimation under error dependence in functional autoregressive models, first was used by Györfi (1989) and then by Masry and Farshiain (1991) Harland (1992), Hardle et al. (1997), Young (1998) and Boske (1998), also most of them have pointed to nonparametric methods in functional autoregressive models. Murana (2001) proposed a semi-parametric approach to short-term forecasting of oil prices and showed how by use of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) and the oil price fluctuations could predict short-term oil price distributions.

The proposed approach can both provide a measure for future oil prices and calculate interval forecasts of immediate oil price. Due to the nonlinear phenomenon of the process which is dominant on the time series of prices, it can be expected that the nonlinear models (such as functional autoregressive models) lead to better predictions. For first-order functional autoregressive models, the estimation of kernel regression function was investigated by Masry (2005) and the application of functional autoregressive models as a functional form of classical models was studied by Cardot and Besse (2005)

Antoniadis (2003) used a small kernel approximation (widely used in various industries and civil engineering) for functional autoregressive models. Also, Zhang (2012) proposed a semi-parametric estimation using the semi-functional partial linear model. But the use of semi-parametric estimation methods only for the first-order autoregressive model with independent error, has been studied by Zhuoxi et al. (2009).

One of the main problems of many macroeconomic models is that the time variables have an inflexible decreasing or increasing form. In addition, it has been proven that in business cycles, the decreasing or increasing trend of key macroeconomic variables such as production and employment during periods of recession is faster than the rate of increase during periods of prosperity. The evaluation of the corporate effectiveness in a special industry is based on a simple idea of dividing the units into efficient units and inefficient units. When analyzing performance, it is necessary to define a Frontier by which to measure the potential inefficiency of the unit. This Frontier is often based on production data (usually labor and capital) - this is the area of technical efficiency. The efficiency of a company can be considered as a situation in which it is not possible to produce more with the given resources. The cost efficiency can be checked by adding information about input cost In analysis. Today, two different approaches are used to measure efficiency: nonparametric and parametric. Nonparametric methods compare the observed inputs and outputs of each firm with firms having a function in a data set without having information about production performance. Stochastic Frontier models by Aigner et al. (1977) is widely used to evaluate the performance of companies in terms of efficiency. In addition to its definite nature, the nonparametric approach is criticized mainly for the fact that efficiency scores are sensitive to the select of inputs and outputs.

Data envelopment analysis method makes the most use of non-parametric methods for model evaluation and optimization. The most common method used to evaluate technical efficiency has been selected regardless of the industry. However, in the models where the percentage of parametric methods in efficiency evaluation is constantly increasing, the models with functional form specifications for the Frontier as well as noise and inefficiency processes are completely parametric. Studies such as Kumbhakar et al. (2007) have tried to reduce some of the limitations in parametric models, but so far all of these approaches have been limited to a univariate response variable. Some researchers (e.g., Simar, 2010; Kuosmanen & Johnson, 2017) have proposed nonparametric estimation of directed interval functions in order to address multiple inputs and outputs, and have raised endogenous issues that are overlooked or are finally considered by imposing restrictive and unacceptable assumptions. In another study, nonparametric methods developed by Simar et al.

(2010) are expanded. Hafner et al. (2018) define multiple inputs and outputs in an almost nonparametric framework while minimizing endogenous problems in the model.

Stochastic Frontier models have been used to predict production, cost, and profit and loss margins models in many papers (Greene, 2005; Kamhakar, 1991; Huang Li, 1994; Mastomarco, 2012;Jin ,2019, Heshmati, 2019; Button, 2009, Zang 2020). Also, the application of stochastic border models are mentioned in the steel industry (Aigner et al., 1977) banking (Simar, 2010), agriculture and in particular the rice production cost model (Button, 2014, 2009) and the stock market (Zubair Hassan, 2005).

In recent years, the researchers in the filed of applied statistics, econometrics and industrial management have done significant works in the form of improving and correcting the estimation of frontier models. So far, many studies have been conducted on the efficiency of the banking system with regard to basic knowledge in stochastic frontier models, and most of them use simultaneous equation methods (Abel 2018). In order examine the differences in competition efficiency between financial actors, differences in product supply in terms of quality and the level of development in financial markets, the frontier models were used and also the bank cost efficiency was evaluated by use of stochastic frontier analysis (Li, 2010).

The growth of theoretical development and improvement of estimation methods for stochastic frontier models is significant due to its high rate of application (Fan, 1996; Kamhakar, 2015 and 2012; Wang, 2015; O'Donnell, 2012). On the other hand, stochastic frontier models are more important due to the role of technical inefficiency and by attention to the input of environmental factor variables and traditional input and output variables.

The selection of estimation method for stochastic frontier model analysis is the maximum likelihood, but the methods of the least squares and non-parametric estimation and so Bayesian estimation developed by Green (2005) have been investigated. Specifically, the maximum likelihood issue for the stochastic frontier model is a finite optimization problem because it has parameters limited to the positive axis. With appropriate initial values, the practical effect of applying such a model may not be effective in predicting, but the use of common approximation methods such as Newton's method and by re-parameterization and interactive standardization of errors will be closer to the appropriate prediction.

In Chen (2014) study, asymptotic estimates and inferences for threshold effects in stochastic frontier models are mentioned. In the present study, structural changes in the model including the threshold function and the threshold parameter value are evaluated using exogenous variable multivariate changes.

In this study, a stochastic frontier model with threshold functional coefficients in different states is investigated. Parametric methods such as maximum likelihood method, conditional minimum squares method have been used and Farnoosh and Mortazavi (2011), Farnoosh and Hajebi (2019) methods have been used for nonparametric moderator estimation method. In the present paper, the efficiency of frontier models is obtained using the semi-parametric method and a more appropriate prediction will be made for the generalities of the model. Also in the present study, some issues like identification and the estimators characteristics are reviewed and them their limited sample performance are examined via Monte Carlo experiments method. The practical implementation of the method is demonstrated using Covid 19 data. The rest of the other sections of this article are as follows. In Section 2, firstly, the threshold autoregressive model and then the threshold autoregressive stochastic frontier model are define in two forms. In this section the proposed model of stochastic frontier with threshold functional coefficients and a semi-parametric approach and with paying attention to the structure of environmental factors are presented.

In the section of Findings, a simulated study is performed to show the behavior of several finite samples of the proposed estimates. In the final section, the conclusion and general discussion and evaluation of the proposed model are presented.

Semi-parametric Estimation in Nonlinear Autoregressive Model with Independent Error

The first-order nonlinear autoregressive model is considered below

$$Y_t = f(Y_{t-1}) + \varepsilon_t \qquad (1\text{-}1)$$

So that the error of the model is a series of independent and identically distributed random variables with zero mean and variance σ^2 .Also Y_{t-1} is independent of ε_t for any t.

Traditionally, a parametric or nonparametric approach can be used to estimate the autoregressive function. If there is information from previous experiments and the analysis is formed under the previous structure, it can be assumed that the regression function related to the nonlinear autoregressive model has a parametric framework as the following parametric model:

$$f(x) \in \{g(x,\theta), \theta \in \Theta\} \qquad (1\text{-}2)$$

Which is an initial choice; where $\Theta \subseteq R^P$ is a parametric space. In this case, the regression function estimator is replaced by the parametric vector estimator θ, and as a result the regression function f(0) is estimated as follows:

$$f(x) = g(x,\theta) \qquad (1\text{-}3)$$

In which θ is an estimator of θ. In other words, if little information is available about the nature of the f(0) function, it is a reasonable non-parametric approach mentioned Fan and Truong (1993), and Francesco (2005). For example, a regression function estimator D, known as a kernel estimator, is defined as follows:

$$f(x) = \frac{\sum k(\frac{y_t - x)}{h_n}.y_t}{\sum k(\frac{y_t - x)}{h_n}} \qquad (1\text{-}4)$$

In which k(0) is a kernel function and h_n is the bandwidth that depends on the number of sample sizes (n).

The kernel estimator is a special case of the second-order polynomial estimator proposed by Hardel and Shibakov (1997). If the parametric assumptions are valid, the parametric method is preferred. However, if the parametric assumptions are not valid, the result of the parametric method can lead to misleading inferences about the regression function. In this case, the nonparametric method is considered without accepting the assumption that the structure of the parameters be controlled with finite dimensions.

Semi-parametric Estimation in First-order Nonlinear Autoregressive Model with Dependent Error

In the present research, the first-order nonlinear autoregressive model with dependent error is defined as follows:

$$Y_t = f(Y_{t-1}) + \varepsilon_t \qquad t \in z$$

$$\varepsilon_t = \rho \quad \varepsilon_{t-1} + u_t \qquad |\rho| < 1$$

(1-5)

In which $\{u_t\}$ is a series of independent and identically distributed variables with mean zero and variance σ^2. Also Y_{t-1} and u_t are independent from each other for each t.

In recent years, a combination of parametric forms and nonlinear functions has been used as a more efficient model in various branches of applied sciences, especially in applied statistics, econometrics and financial studies. For example, semi-parametric models with a single index and in form of generalized linear partial time series models are one type of these combinations. Time series models with structure of nonlinear functions have been proposed in Hidalgo (1992), Delgado and Robinson (1992), Truong and Stone (1994). They have considered a time series model with non-parametric errors.

$$Y_t = x_t'\beta + u_t \qquad (1\text{-}6)$$

If the model error is defined as follows:

$$u_t = g(u_{t-1}) + \varepsilon_t \qquad (1\text{-}7)$$

x_t is also a static time series with finite second torque. In such models, u_t, Y_t are scalar and defined as numerical observations and and also g(0) is an unknown function. Also $\{u_t\}$ is a static time series with zero mean and finite variance. Errors ε_t are random variables with the same distribution and independent of each other. Schick (1996) proposed an efficient estimation in a semi-parametric collective regression model with autoregressive error. Truong and Stone (1994) proposed nonparametric regression models with linear autoregressive error as follows:

$$Y_t = g(x_t) + u_t$$

The model error is defined as follows:

$$u_t = \theta u_{t-1} + \varepsilon_t$$

In such models, (x_t, y_t) is a two-variable time series and θ is an unknown parameter with condition $|\theta| < 1$.

As in the previous model, $g(.)$ is an unknown function and $\{\varepsilon_t\}$ is a sequence of independent errors with mean zero and finite variance. In order to estimate the parameters, they used non-parametric methods (Truong and Stone, 1994). In financial studies with hazard fluctuations and with unstable conditional variance, those models with dependent errors are used which are presented in Tong (1990) and Li (1999) articles. From 1995 to 2005, several linear autoregressive conditional hierarchical models in which the error is dependent have been introduced, so that one of such econometric models is as follows:

$$Y_t = g(Y_{t-1}) + \beta Y_{t-1} + \varepsilon_t$$

(1-8)

In which $\{\varepsilon_t\}$ is a static time variable and $\sigma^2(y) = E\left[\varepsilon_t^2 \;\middle|\; Y_{t-1} = y\right]$ is a smooth function of y. Both components of the model, parameter β and function g, can be estimated which in the articles conducted by Granger et al. (1997); Hjellvik (1998); Gao and King(2005) some studies have been examined in this filed. Zhuoxi et al. (2009) have successfully introduced a modified model of it in the form of a first-order nonlinear time series model, as follows, and have estimated the regression functions.

They used semi-parametric estimation methods in order to estimate the $f(.)$ function; so that the errors of this model were random variables independent of each other, identically distributed and had a mean of zero and variance σ^2. It should be noted that in this study the suggested model has been proposed with the dependent error, which is in the form of a first-order autoregressive model. In this regard, $f(.)$ is considered as a parametric framework known as the parametric model, so:

$$f(x) \in \{g(x,\theta); \theta \in \Theta\}$$

Or

$$f(x) \in \left\{g^*(x).h(\theta); \theta \in \Theta\right\} \qquad (1\text{-}9)$$

In which: $\Theta \subseteq R^p$ forms the parametric space.

The regression function $f(.)$ is defined by the conditional least squares method as $\hat{f}(x) = g(x, \hat{\theta})$ or $\hat{f}(x) = g^*(x).h(\hat{\theta})$, provided that $\hat{\theta}$ is an estimator of θ. In other words, if information about the nature of $f(.)$ is available (some information about behaviors and changes in the $f(.)$ function is obtained experimentally), the nonparametric method would be appropriate.

A semi-parametric estimator based on the Nadaraya-Watson method, used as a regression function estimator $f(.)$, is a kernel estimator described below:

$$\hat{f}(x) = \frac{\sum_{j=1}^{n} k\left(\frac{y_{j-1} - x}{h_n}\right) Y_j}{\sum_{j=1}^{n} k\left(\frac{y_{j-1} - x}{h_n}\right)}$$

(1-10)

In which $k(.)$ and h_n are the Kernel function and the corresponding bandwidth, respectively.

1-4- Stochastic Frontier Model with Smooth Transfer Function

A production stochastic frontier model with threshold functional coefficients in normal (non-autoregressive) form is introduced as follows, in which C is the logarithm of the input variables in the structure of the production process. $Xt' = [X_{it},...,X_{ut}]$ is a logarithmic vector of k input variables. Z_t is a $P \times 1$ vector (for example, the time or location of two indices can also be used as a spatial time series). The simplest form frontier model of smooth transfer function is as follows:

$$y_t = \alpha(z_t) + x_1'(\beta_1(z_t)\gamma_m + \beta_2(z_t)(1-\gamma_m)) + v_t - u_t$$
$$y_t = \theta(z_t) + x_1'(\beta_1(z_t)\gamma_m + \beta_2(z_t)(1-\gamma_m)) + \varepsilon_t$$
$$\theta(z_t) = \alpha(z_t) - E(u_t)$$
$$\varepsilon_t = v_t - (u_t - E(u_t))$$
$$\gamma_m = Iz_t(\gamma) = \begin{cases} 1 & y_m > \gamma \\ 0 & y_m < \gamma \end{cases}$$

(1-11)

$\alpha(.)$ is the width of the origin and $\beta(.)$ is a $k*1$ vector of the parameter and xt is a independent variable vector of the input logarithm k which is defined as a threshold and piecewise function. Therefore all three coefficients of the model, $\beta_1(.), \alpha(.), \beta_2(.)$ are placed as normal and threshold in the unknown function of z_t. Also $V_t \overset{i.i.d}{\approx} N(0, \sigma_v^\tau)$ are the model perturbations. $u_t = U(z_t, \delta)$ is a positive technical inefficiency.

According to Caudill (1995), the perturbation function is defined hierarchically. That is, it is assumed that $\sigma_v(z_t) = \exp\{\delta_0 + \delta_1' z_t\}, u_t = \sigma_u(z_t)\lambda_t$ and $\lambda_t \overset{iid}{\sim} N^t(0,1)$ should be $v_t \sim iidN^+(0, \sigma_u^2(z_t))$ equivalent so that $\sigma_u^2(z_t) = \exp\{2(\delta_0 + \delta_1' z_t)\}$ be recognized. Functional form and distributive hypotheses are required. By guaranteeing that u_t is positive and considering the above hypotheses, it can be shown:

$$\mathrm{E}[u_t] = \sqrt{\frac{2}{\pi}}.\sigma_u(z_t) = \sqrt{\frac{2}{\pi}}\exp(\delta_0 + \delta_1' z_t)$$

(1-12)

$$\lambda = \frac{\sigma_u}{\sigma_v} \Rightarrow \sigma_u = \lambda.\sigma_v$$
$$\sigma^2 = \sigma_u{}^2 + \sigma_v{}^2 \Rightarrow \sigma_v{}^2 = \sigma^2 - \lambda^2.\sigma_v{}^2$$
$$\sigma^2 = \sigma_v{}^2(1+\lambda^2) \Rightarrow \sigma_v{}^2 = \frac{\sigma^2}{(1+\lambda^2)} \Rightarrow \sigma_u = \frac{\lambda.\sigma}{\sqrt{(1+\lambda^2)}}$$
$$\sqrt{\frac{2}{\pi}}.\sigma_u = \sqrt{\frac{2}{\pi}}.\frac{\lambda.\sigma}{\sqrt{(1+\lambda^2)}} \Rightarrow$$

The production border in relation (1-12) is not conditional expectation of y_t, because the error analysis sentence $V_t - U_t$ does not have a mean of zero. To solve this problem, the Equation (1-11) can be rewriten and be converted into another conventional form:

$$y_t = \alpha(z_t) + x_t'(\beta^*(z_t, \gamma_m) + v_t - (u_t - E(u_t) - E(u_t))$$

So that:

$$\beta^*(z_t, \gamma_m) = \beta_1(z_t).\gamma_m + \beta_2(z_t).(1-\gamma_m)$$
$$\gamma_m = I_{y_m}(\gamma) = \begin{cases} 1 & y_m \geq \delta \\ 0 & y_m < \delta \end{cases}$$

Or in equivalent;

$$y_i = \theta(z_t) + X_t'\beta^*(z_t, \gamma_m) + \varepsilon_t \qquad 3$$

In which:

$$\varepsilon_t = v_t - (u_t - E(u_t))$$
$$\theta(z_t) = \alpha(z_t) - E[u_t]$$

Equation (3) can always be estimated based on a stochastic smooth coefficient model and is defined as follows (Huang Li, 2002):

$$W_t' = [1 - X_t'] \qquad \hat{\rho}(z_t) = (\hat{\theta}(z_t), \hat{\beta}*(z_t, \hat{\gamma}_m)$$

Other smooth coefficient estimates can be written as follows:

$$\hat{\rho}(z_t) = \left[\sum_{j=1}^{n} W_j W_j' \; k\left(\frac{z_j - z_t}{h}\right)\right]^{-1} \sum W_j y_j k\left(\frac{z_j - z_t}{h}\right)$$

(1-13)

In which, n is the sample size. $K(0)$ is the production kernel and h is a $p-$ vector of bandwidth that can be selected by the least squares validation method. Therefore:

$$\varepsilon_t = y_t - \hat{\theta}(z_t) - X_t^{'}\beta^{*}(z_t, \hat{\gamma}_m)$$

This is the first step of estimation in the initial model.

Given $\varepsilon_t = V_t - U_t + E[u_t]$ where $E[U_t] = \sqrt{\frac{2}{\pi}}\sigma_u(z_t); U_t = \sigma_u(z_t).\lambda_t$ is. Form u_t can be placed in the above equation.

$$\hat{\varepsilon}_t = \sqrt{\frac{2}{\pi}}.\delta_u(z_t) + v_t - \sigma_u(z_t) - \gamma$$

$$= \sqrt{\frac{2}{\pi}}\exp(\delta_0 + \delta_1^{'} z_t) + v_t - \exp(\delta_0 + \delta_1^{'} z_t)\lambda_t$$

where in

$$\sigma_u^2(z_t) = \exp[2(\delta_0 + \delta_1 z], \Omega_t = \frac{\sigma_u(z_t)}{\sigma_v}$$

$$\sigma_t^2 = \sigma_v^2 + \sigma_u^2(z_t) = \sigma_v^2 + \exp[2(\delta_0 + \delta_1^{'}(z_t)]$$

That $\sigma_v^2, \sigma^2{}_u$ can be obtained first by the maximum likelihood. $\sigma_u^2(z_t), \Omega_t$ can be re-estimated.

Least Squares Method

Weight method can be used as:

$$h_n(z, \theta_1, \theta_2, r) = \sum_{t=1}^{n} W_t(z_t)\big(y_t - g(y_{t-1}, \theta_1).\alpha(r,t) - g_2(y_{t-1}, \theta_2).(1-\alpha(r,t))$$

$$- g_2(y_{t-1}, \theta_2) - (1-\alpha(r,t))^2$$

(1-14)

So that: J is a function of weight so that K given that:

$$y_t = g_1(y_{t-1}, \theta_1).\alpha(r,t) + g_2(y_{t-1}, \theta_2).(1-\alpha(r,t))$$
$$+ h_n(z, \theta_1, \theta_2, r) + \varepsilon_t$$

$$\tilde{V}_1(\theta_1, \theta_2, r) = y_t - g_2(y_{t-1}, \theta_1).\alpha_r, t)$$
$$- g_2(y_{t-1}, \theta_2).(1-\alpha(r, t_1) - \sum W_t(z_t)$$
$$- (y_t - g_1(y_{t-1}, \theta_1).\alpha(r,t) - g_2(y_{t-1}, \theta_2)$$
$$(1-\alpha(r,t))$$

As a result, the θ estimator contains r, θ_2, θ_1 (threshold value) with method LS as follows:

$$\hat{\theta} = \arg\min \tilde{V}_1(\theta_1, \theta_2, r)$$

In fact, $\hat{r}, \hat{\theta}_2, \hat{\theta}_1$ is the total conditional least squares (CLS) based on $y_n, ..., y_2, y_1$ observations. Under some conditions of the technical estimator, Klimko and Nelson (1978) showed strong CLS adaptation under different conditions. Next, the nonparametric modifier estimation in the form of $\hat{g}_1(x, \hat{\theta}_2).\varepsilon_1(x)$ and $\hat{g}_2(x, \hat{\theta}_2).\varepsilon_2(x)$ is performed using the fitting criterion of native polynomials $L^2 - fitting$.

According to equation (*)

$$y_t = f_1(y_{t-1}).\alpha(r,t) + f_2(y_{t-1}).(1-\alpha(r,t)) + h_n(z_t) + \varepsilon_t$$

is put instead of $h_n(z_t)$:

$$h_n(z, \hat{\theta}_1, \hat{\theta}_2, r) = \sum W_t(z_t)((y_t - g_1(y_{t-1}, \hat{\theta}_1)$$
$$\alpha(\hat{r}, t) - g_2(y_{t-1}, \hat{\theta}_2).(1-\alpha(\hat{r}, t_1))^2$$

(1-15)

If the parametric assumptions be established in the relation

$$** \quad \begin{aligned} f_1(x) &\in \{g_1(x, \theta_1), \theta_1 \in \Theta\} \\ f_2(x) &\in \{g_2(x, \theta_2), \theta_2 \in \Theta\} \end{aligned}$$

Then the parametric method is acceptable for a number of reasons.

However, if the parametric assumptions of relation (**) are not established, then the nonparametric method leads us to a misleading inference about the regression function.

In this case, the nonparametric method can be performed without accepting these assumptions, thus making it possible to introduce an approach that includes both parametric and nonparametric methods. In this comparison, by combining the methods of Fan (1996), Farnoosh and Mortazavi (2011) and Farnoosh et al. (2019) in frontier models have been followed.

In the combined method, the initial estimation of the parameters and the introduction of the non-parametric function in the frontier model are used. For this purpose, for the nonparametric estimation section, using the same idea as Hjort and Jones (1996), Naito (2004), the criterion for fitting second-order polynomials is defined as follows:

$$y(x,\varepsilon_1,\varepsilon_2)=\frac{1}{h_n}\int k(\frac{t-x}{h_n})\left[f_1(t)-g_1(t,\hat{\theta}_1).\hat{\varepsilon}_1(t)\right]^2$$

$$+\frac{1}{h_n}\int k\left(\frac{t-x}{h_n}\right)\left[f_2(t)-g_2(t,\hat{\theta}).\hat{\varepsilon}_2(t)dt\right]^2$$

(1-16)

In which, $\hat{f}_i(x)$ are the unknown autoregressive functions. The estimator $\hat{\varepsilon}_i(x)$ is obtained from $\varepsilon_i(x)$ by minimizing the above second-order fitting criteria relative to $\xi(x)$. Thus nonparametric estimators $\varepsilon_i(x)$ are obtained.

The above formula is focused on the $\tilde{f}_i(x)$ in order to estimate the nonlinear autoregressive function, and therefore there is no unknown value in the estimate of $f(x)$, which is represented by $\tilde{f}(x)$, and can be be obtained using sample information and data simulation. The asymptotic efficiency and parameters are confirmed by the following conditions according to the articles of Farnoosh and Mortazavi (2011), Farnoosh and Hajebi (2016). In order to estimate the nonparametric moderating factor, a criterion called localized fit L2 is used and the parameters are estimated with the initial conjecture for the g (.) function, and taking into account the Taylor series, and thus the g (.) function is finally estimated. For the compatibility of the model parameters, the following items are considered as classical hypotheses:

- The sequence of the random variable yt is an ergodic, static, continuous, bounded and monotonically increasing sequence.
- The functions g1, g2 both are continuous mathematical functions and has continuous derivatives.
- The mathematical expectation of difference is a random variable of yt and the function g is finite.
- yt is a random mixing variable.

Results of Practical Example

In order to show the efficiency of stochastic estimation for the frontier model with a smooth transfer function, the error investigation approach has been considered in the present study. Using Monte Carlo simulations with known nonlinear functions, the error of parameters estimation in the nonlinear function is investigated, which good results has been shown. In the practical example using E-views software, the proposed model designed for the rate of patients recovery based on the variables of the rate of patients with confirmed testing and mortality rate and the number of critically ill patients is considered that the model has good results.

Dependent Variable: SHEDATBIMARI
Method: ARDL
Date: 06/22/20 Time: 00:02
Sample (adjusted): 3/17/2020 5/15/2020
Included observations: 60 after adjustments
Maximum dependent lags: 4 (Automatic selection)
Model selection method: Akaike info criterion (AIC)
Dynamic regressors (4 lags, automatic): NERKHBEBOD NERKHFOTI TESTED MOBTALAYAN
Fixed regressors: C @TREND
Number of models evalulated: 2500
Selected Model: ARDL(4, 4, 1, 4, 4)

Prob.*	t-Statistic	Std. Error	Coefficient	Variable
0.0000	6.690178	0.118507	0.792834	SHEDATBIMARI(-1)
0.4504	0.762865	0.163623	0.124822	SHEDATBIMARI(-2)
0.0002	-4.134554	0.168796	-0.697895	SHEDATBIMARI(-3)
0.0002	4.063472	0.116376	0.472892	SHEDATBIMARI(-4)
0.7519	-0.318441	385.9949	-122.9167	NERKHBEBOD
0.4569	-0.751894	376.3379	-282.9663	NERKHBEBOD(-1)
0.5316	0.631533	383.0327	241.8978	NERKHBEBOD(-2)
0.1341	2.531765	293.6553	449.8110	NERKHBEBOD(-3)
0.0429	-2.096994	274.6742	-575.9904	NERKHBEBOD(-4)
0.0504	2.022386	73.81768	149.2879	NERKHFOTI
0.0167	3.202820	69.41803	83.49742	NERKHFOTI(-1)
0.2240	1.236711	0.003033	0.003750	TESTED
0.1056	-1.658666	0.004420	-0.007331	TESTED(-1)
0.0333	2.211643	0.004200	0.005089	TESTED(-2)
0.6403	0.471152	0.003999	0.001884	TESTED(-3)
0.0781	-1.811989	0.003031	-0.005491	TESTED(-4)
0.0089	2.763870	0.056967	0.157450	MOBTALAYAN
0.5763	-0.563801	0.073466	-0.041420	MOBTALAYAN(-1)
0.0342	-2.988385	0.079664	-0.158403	MOBTALAYAN(-2)
0.9349	-0.082249	0.085018	-0.006993	MOBTALAYAN(-3)
0.0010	3.563985	0.067373	0.240116	MOBTALAYAN(-4)
0.4527	0.758860	743.6806	564.3495	C
0.2911	1.071044	15.27525	16.36046	@TREND

3170.617	Mean dependent var	0.993070	R-squared
564.0710	S.D. dependent var	0.988949	Adjusted R-squared
11.28624	Akaike info criterion	59.29721	S.E. of regression
12.08907	Schwarz criterion	130097.9	Sum squared resid
11.60027	Hannan-Quinn criter.	-315.5873	Log likelihood
2.135360	Durbin-Watson stat	240.9950	F-statistic
		0.000000	Prob(F-statistic)

*Note: p-values and any subsequent tests do not account for model selection.

Conclusion

In the present study, which introduces a threshold autoregression model in order to estimate the stochastic parameters of a frontier model, nonparametric modulating methods and using maximum likelihood estimation methods, a useful experiment in parameter detection has been used. In the application part, the proposed model has a good predictability for the recovery rate of COVID- 19 virus.

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