

A Multilevel Analysis of Saudi Arabian Student 8th Grade Mathematics Achievement TIMSS 2011

Maha Al-Mutairi^{1*}, Khaled Bennour², Shuruq Hasen Alotaibi³

^{1*} Department of Mathematics, Shaqra University, Saudi Arabia, malmataery@su.edu.sa

² Department of Statistics and Operations Research College of Science, King Saud University, Saudi Arabia, kbennour@ksu.edu.sa

³ Department of Mathematics, Shaqra University, Saudi Arabia, Shalotaibi@su.edu.sa

Abstract:

This article uses the hierarchical linear modelling (HLM) technique to explain causally the Mathematics Performance achievement of students in Saudi Arabia. Particularly, the HLM technique was applied to the TIMSS 2011 data set where five variables (Home educational resources (BSBGHER), Like learning mathematics (BSBGSLM), Self-confidence in mathematics (BSBGSCM), Engaged in mathematics learning (BSBGEML), Value learning math (BSBG SVM)) at the student level and three variables (Emphasis on academic success (BCBGEAS), School discipline and safety (BCBGDAS), and Instruction affected by mathematics resources shortages (BCBGMRS) at the school level, were used to build the hierarchical linear model so as to predict the status of mathematically 8th grader. The final model suggested that all the student level factors are found to be significant but their impact on achievement do not vary significantly across the population of schools, i.e. BSBGHER, BSBGSLM, BSBGSCM, BSBGEML, and BSBG SVM significantly predicted the status of Mathematics Performance achievement of students in Saudi Arabia. At the school level, it is found that BCBGDAS and BCBGMRS have a significant impact on performance. However, a scale point increase in the availability of school resources for mathematics decreased achievement by 4 points.

Keywords: Hierarchical linear model; Mathematics achievement; TIMSS 2011.

1. Introduction

It is well known that mathematical performance achievement is influenced by various factors, such as educational, psychological, biographical, social, among others. These factors can be categorized as students and school variables. In this article, we consider to the TIMSS 2011 data set to investigate the mathematical performance achievement in Saudi Arabia students using five student level variables and three school level variables.

Considerable studies have been done to investigate trends in mathematics achievement and the factors effecting mathematics learning and performance—e.g., [1-2,15,21,25,26,35]. For example,

[21] investigated the factors of mathematics achievement including students' gender, age, ethnicity, their family socioeconomic status and school characteristics. In [26] the effects of school, students' attitudes and their beliefs in mathematics learning on students' performance were studied. Mathematics beliefs and self-concept were also considered in [15] and [36]. While [2] studied gender differences in mathematics achievement among high-school students.

Student engagement is another important factor which is defined as the level of participation, and intrinsic interest that a student shows at the school. It relies on students' behaviour at their

schools such as persistence, effort, motivation, positive learning values, enthusiasm, and interest, see [8]. Various studies have displayed that student engagement

is fundamentally essential in promoting achievement. This because engaging students during the learning process leads to success and more learning, both inside and outside school (e.g., [27,33]).

The literature shows a variety of studies in which gender differences in mathematics were considered, see [7,12,18,30], just to mention a few. These studies have demonstrated that gender differences in mathematics performance can be regarded as small in many countries. Furthermore, these studies provide evidence that the magnitude of gender differences has declined compared to previous decades. [7] analyzed two data sets from the TIMSS 2003 and the PISA 2003 studies to check cross-national patterns of gender differences in mathematical achievement, attitudes, and affect and assessed the links of these patterns to gender equity at the national scale. The results of this study demonstrated that the gender gap in mathematics continues in some countries. Despite the similarities between boys' and girls' achievements, it appears that boys feel more confident and less anxious in their mathematical abilities than girls. In addition, boys are more extrinsically and intrinsically to do well in mathematics than girls, which is closely consistent with related research results in the literature (e.g., [18]). Also, boys scored one third of a standard deviation higher than girls on mathematics self-concept and self-efficacy [7].

The effect of students' SES (e.g., parents' education and home educational resources) on achievement in science were excessively investigated [5,9,22,23,31,34,37]. These studies demonstrated that students from homes where their parents have a higher level of education and have more educational resources tend to perform better in science in comparison to those students their parents have lower levels of education and have less educational resources. [6] used a multilevel modelling technique to study students'

achievement. The results revealed that family background characteristics accounted for 68.33% of the total variance in students' achievement [4].

Developing a positive attitude in mathematics learning has been excessively recognized. Indeed, many countries have set it as one of the main goals of mathematics education at schools. For example, in the case of Singapore, the national mathematics curriculum states "mathematics education aims to enable pupils to develop positive attitudes towards mathematics, including confidence, enjoyment and perseverance" (Ministry of Education [MOE], 2000, p. 9). The academic emphasis of school is another essential variable in explaining student achievement. Setting high academic targets for students leads to suitable learning environment which motivates students to work hard and higher academic achievements [17]. Literature that is closely related to the relationship of academic emphasis and achievement leads to consistent all levels of education's results, i.e., elementary, middle, and high school, academic emphasis and achievement were positively related, even controlling for socioeconomic factors (see, [10,16]).

The school discipline and safety characteristics explain the variance in student achievement among schools. At schools in which the disciplinary climate is strong, students usually perform better both behaviourally and academically [20]. There are many studies addressing the influence of school safety conditions on student's achievement. Based on these studies, violence has been found to hinder cognitive, social, and emotional development [28]. In violent schools, it appears that students have less time to focus on academic activities as they are paying more attention to other factors and personal safety issues, see [3,28]. So, it can conclude that unsafe school conditions have a negative impact on students' academic achievements. The relationship between school resources, (e.g., textbooks, computers, calculators, the number of pupils per teacher) and student achievement is one of the most debated issues in education which is of particular interest to policy-makers who are responsible for making decisions regarding the allocation of resources to schools. There are inconsistent results

about the relationship of school resources and academic achievement. While there are studies which concluded that there is no strong and continuous link between school resources and the academic performance of students (see,[13]), some studies showed that expenditures per student had a relatively large degree of positive effect on the academic performance of students [14]. Good attendance at school (GAS) is the evaluation of the school principals concerning the seriousness of students' behaviour which is measured by: arriving late at school, skipping class and absenteeism. The findings, internationally, show that the average achievement is higher in schools where these problems are not serious (see, [22]).

A cross-national study indicates that GAS accounted for a slight portion of the variance in the achievement of Singaporean eighth-graders ([19]), indicating that these problems are not serious among the Singaporean schools. In the 2007 TIMSS, Singapore, with only 4% of schools, was one the lowest countries having serious problem with students' behaviour. However, the achievement average of this group of schools was lower by 211 scale points compared to schools where the students' behaviour was not serious.

This article aims to investigate the factors that affect the mathematics performance achievement of students in Saudi Arabia by using of HLM analysis to TIMSS 2011 data [1]. HLM is a comprehensive statistical technique for analysing hierarchical structures such as students nested within schools [21,32]. Through this approach, the factors that influenced Mathematics Performance achievement are examined from both student and school perspectives. Undoubtedly, identifying which factors influence students' academic achievement is important to educational stakeholders, especially for educational decisionmakers who can make use of these findings to guide both policy and practice. The specific research questions for this study are formulated as follows:

1. How do 8th grade students' mathematics performance achievement vary between students within school and across schools?
2. What factors at the student level significantly contribute to

influencing students' Mathematics Performance achievement?

3. As well as controlling for student variables, what factors at the school level significantly contribute to influencing of students' mathematics achievement Performance?

The rest of the paper is structured as follows. First, we describe the Methods and the

TIMSS 2011 Saudi Arabia data, including all Student Level (BSBGHER, BSBGSLM, BSBGSCM, BSBGEML and BSBGSVM) variables and School Level ((BCBGEAS, BCBGDAS and BCBGMRS) variables. Also, the descriptive statistics of all of such variables is presented. Second, two-level HLM analysis is used to estimate the model's parameters. Finally, we state our conclusion.

2. Materials and Methods

2.1 Sample Design

TIMSS has been designed to give a valid and reliable measurement of trends in student achievement over different countries, which keeps a minimum the load on schools, teachers, and students. The program employs strict school and classroom sampling methods such that achievement in the student population can be accurately estimated by assessing only a sample of students from a sample of schools. TIMSS assesses the achievement in mathematics and science for two particular grade levels and so TIMSS has a target population, namely, all students enrolled in the eighth grade. Note that countries may assess both student populations or one of them. As an assessment of reading comprehension in primary school, the target population for PIRLS is all students enrolled in the eighth grade. This means that TIMSS employs a two-stage random sample design, with a sample of schools drawn as a first stage while intact classes of students chosen from each of the sampled schools as a second stage. Since TIMSS pays particular attention to students' curricular and instructional experiences, and these typically are organized on a classroom basis, intact classes of students

are sampled rather than individuals from across the grade level or of a specific age.

Stratification: Stratification is the process of categorizing schools in a target population into different groups, or strata, based on shared features such as geographic location or school type. Region of the country (e.g., states or provinces); school type or source of funding (e.g., public or private); language of instruction; level of urbanisation (e.g., urban or rural area); socioeconomic indicators; and school performance on national examinations are examples of stratification variables used in TIMSS. There are two types of school stratification: explicit and implicit. Explicit stratification involves creating a distinct school list or sampling frame for each stratum and selecting a sample of schools from that stratum. The main rationale for considering explicit stratification in TIMSS is the uneven allocation of the school sample across strata. For example, explicit stratification by area may be used to assure the same number of schools in the sample for each region, regardless of the relative population size of the regions, in order to obtain equally valid estimates for each geographic region in a region.

School Sampling Frame

The sampling frame is a list of all schools in the country with students in the target grade, and it is from this list that the school sample is selected. A well-constructed sampling frame ensures that the national target population is completely covered without being tainted by inaccurate or duplicate entries, or entries that relate to components that are not included in the intended target population. If the national sample strategy asks for explicit stratification, each explicit stratum should have its own sampling frame. Because the size of the school impacts its chance of selection, a proper school measure of size (MOS) is an important part of the national sampling plan. The most accurate assessment of a school's size is an up-to-date count number of students in the target grade. If the target grade's number of students isn't accessible, the total number of students enrolled in the school may be the best alternative.

Sampling Schools

Once the school sampling frame has been structured, sample schools can be then drawn. Because the sampling frame is explicitly stratified; one needs to decide the number of schools to be sampled in each stratum (i.e., number of schools sampled is to be allocated among the explicit strata). Then a sample of schools is selected within each explicit stratum based on systematic sampling method with probabilities proportional to size.

Sampling Classes

All classes with students from each sampled school at the target grade are listed, and with equal probability intact (one or more) classes are selected using systematic random sampling. This process is carried out by using the WinW3S sampling software. The selection of classes, combined with the PPS sampling approach for schools, in general results in a self-weighting student sample. For the case when the school has multi-grade classes (i.e., the class consists of students from more than one grade level), only students from the target grade are eligible to be sampled.

Sampling Weights: The sampling weights includes School, class, and students

Weighting Component. For sampled schools in TIMSS (which is sampled with probability proportional to school size), the basic school weight for the i th sampled school is given by:

$$w_i = \frac{N}{n_i}$$

Here N represents the number of all sampled schools, n_i represents the measure of size for the i th school, and

$$n_i = \sum_{j=1}^J n_{ij}$$

Class and students Weighting Component:

The class weighting component represents the class within each school selection chance. Once a specific school is sampled and has considered to participate in TIMSS from the list of all classes in the sampled school at the target grade, one or two classes will be sampled with equal probability. Note that since large schools have more classes from which to sample

compared with those smaller schools, then the probability of class selection varies depending on the size of the school, and students in small schools are more likely to have their class selected than those students in large schools. This means the relatively greater selection probability candidates in small schools balance their lower selection probability during the first stage, as probability-proportional-to-size school sampling gives in higher selection probabilities for larger schools.

The student weighting component consists of the student-within-class selection probability. Once intact classes are specified, then all students in the class will be included, and so this probability is known as a unity. However, under some circumstances, students might be sampled within the class, with a probability lower than unity. The basic class-within school weight for a specific class is defined as the inverse of the probability of the class that has been selected from all of the classes belonging to its school. While the basic student weight is the inverse of the probability of a student in a sampled class being selected.

Under equal probability sampling, the basic class weight () for sampled classes in the i^{th} school is defined as:

$$= \frac{1}{n_i} ,$$

where is the total number of eligible classes, and (takes values 1 or 2) is the number of sampled classes. For an intact class with no student subsampling, the basic student weight for the j^{th} class in the i^{th} school can be computed by: $w_{ij} = 1$, while for classes with student subsampling, the basic student weight for the j^{th} class in the i^{th} school can be computed by:

$$w_{ij} = \frac{1}{n_{ij} + 1} ,$$

where ' is the number of students in the j^{th} class of the i^{th} school selected, and

' is the number of students in the class that are not selected.

Note that all student data listed in the TIMSS international reports are weighted by the so-called overall student sampling weight (or TOTWGT in the TIMSS international databases). There are other factor were considered during the sampling design and process including Participation Rates: weighted and unweighted for both school and students participation rates, overall weighted participation rate. For more details we refer the reader to structure pf TIMSS 2011 data set.

The following Table shows the number of Schools and students sample sizes for 8th grad in Saudi Arabia that are considered in the analysis of this article.

Table 1. Details of school and students for 8th grad in Saudi Arabia				
Schools in Original Sample	Eligible Schools in Original Sample	Schools in Original Sample that Participated	Replacement Schools that Participated	Schools that Participated
154	153	150	3	153
Within-school Student Participation	Sampled Students in Participating Schools	Students Withdrawn from Class/School	Students Excluded	Eligible Students
98%	4,477	35	3	4,439

This study uses the TIMSS 2011 Saudi Arabia data. Various explanatory variables at the student level and the school level are used. The outcome variable for this study is students' Mathematics Performance achievement in TIMSS 2011. TIMSS 2011 employed five plausible values to estimate the Mathematics Achievement of each

student. At the student level, a total of five variables are included in the analyses namely, Home educational resources (BSBGHER), Like learning mathematics (BSBGSLM), Self-confidence in mathematics (BSBGSCM), Engaged in mathematics learning (BSBGEML), Value learning math (BSBG SVM). At the school level, a total of three variables

was included in the analyses extracted from the Saudi Arabia school database. These variables are: Emphasis on academic success (BCBGEAS), School

discipline and safety (BCBGDAS), Instruction affected by mathematics resources shortages (BCBGMRS).

Table 2. Explanations for Independent Variables at student and school levels.

Students Level	
Variable	Description
Home educational resources (BSBGHER)	This scale is based on 8th-grade students' responses to the following variables: number of books in the home; educational aids in the home (computer, study desk/table for own use, dictionary); and parents' education (mother and father) [1=few resources, 2=some resources, 3=many resources].
Like learning maths (BSBGSLM)	Students like learning mathematics: The scale was created by TIMSS and based on students' responses to the following five statements: a) I enjoy learning mathematics; b) I wish I did not have to study mathematics; c) Mathematics is boring; d) I learn many interesting things in mathematics; e) I like mathematics [1=don't like learning math's, 2=somewhat like learning math's, 3=like learning math's].
Self-confidence in maths (BSBGSCM)	Students' confidence in mathematics: The scale was created by TIMSS and based on students' responses to the following seven statements: a) I usually do well in mathematics; b) Mathematics is harder for me than for many of my classmates; c) I am just not good at mathematics; d) I learn things quickly in mathematics; e) I am good at working out difficult mathematics problems; f) My teacher tells me I am good at mathematics; g) Mathematics is harder for me than any other subject [1=not confident, 2=somewhat confident, 3=confident].
Engaged mathematics learning (BSBGEML)	Engaged mathematics learning: The scale was created by TIMSS and based on students' responses to the following five statements: a) I know what my teacher expects me to do; b) I think of things not related to the lesson (reverse coded); c) My teacher is easy to understand; d) I am interested in what my teacher says; and e) My teacher gives me interesting things to do [low=1, medium=2, high=3].
Value learning math (BSBG SVM)	Students' value in mathematics: The scale was created by TIMSS and based on students' responses to the following six statements: a) I think learning mathematics will help me in my daily life; b) I need mathematics to learn other school subjects ; c) I need to do well in mathematics to get into the university of my choice ; d) I need to do well in mathematics to get the job I want ; e) I would like a job that involves using mathematics; f) It is important to do well in mathematics [value =1, Somewhat Value =2, Do Not Value=3]
School level	

Emphasis on academic success (BCBGEAS)	School emphasis on academic success: The index was created by TIMSS and based on students' responses to the following five statements given by school principals: a) Teachers' understanding of the school's curricular goals; b) Teachers' degree of success in implementing the school's curriculum; c) Teachers' expectations for student achievement; d) Parental support for student achievement; and e) Students' desire to do well in school [1=medium, 2=high, 3=very high].
INSTRUCTION AFFECTED BY MATHEMATICS RE-SOURCE SHORTAGES (BCBGMRS)	School resources: The index was created by TIMSS and based on principals' responses related to how much capacity is available to provide instruction affected by a shortage or inadequacy of the following statements: Instructional materials (e.g., textbooks); Supplies (e.g., papers, pencils); School buildings and grounds; Heating/cooling and lighting systems; Instructional space (e.g., classrooms); Technologically competent staff; computers for instruction; Teachers with a specialization in mathematics; Computer software for mathematics instruction; Library materials relevant to mathematics instruction; Audio-visual resources for mathematics instruction; Calculators for mathematics instruction [1= affected a lot, 2=somewhat affected, 3=not affected].
School discipline and safety (BCDGDAS)	The index was created by TIMSS and based on students' responses to the following Discipline and five statements: a) This school is located in a safe neighborhood; b) I feel safe at this school; c) This school's security policies and practices are sufficient; d) The students behave in an orderly manner; and e) The students are respectful of the teachers [1=moderate problems, 2=minor problems, 3=hardly any problems].

Table 3. gives descriptive statistics (Mean and Std. Deviation) of Student Level and School Level variables for Saudi Arabia in TIMSS 2011.

Table 3. Descriptive statistics of Student Level and School Level variables for Saudi Arabia in TIMSS 2011.

Level	Variable	Mean	Std. Deviation
Student Level	Home educational resources	9.35	1.96
	Student like learning mathematics	10.05	2.10
	Student ENGAGED with mathematics	10.35	1.90
	Student value learning mathematics	10.17	2.00
	Student CONFIDENCE with mathematics	10.61	1.98
School Level	School discipline and safety	9.68	2.59
	School emphasis on academic success principal reports	9.89	2.20
	Instruction affected by mathematics resources shortages	9.33	1.39

3. Data Analysis

HLM is used for two-level HLM analysis (see [29]) because of the nested structure of the data and the sample design. A model building process is applied to examine the likelihood of the selected student and school variables in influencing the students' Mathematics Performance achievement modelled for Saudi Arabia students. The model building process involved the inclusion and examination of student level variables (Level 1) and followed by testing the direct and moderating effects of school level (Level 2) variables on the criterion variable.

First, the proportions of variance of student Mathematics Achievement Performance at the student and school levels are examined (i.e., fully unconditional model—Model A). Second, student variables were added to the model as Level 1, and non-significant variables are removed (model trimming). This resultant model (Model B) answers the question to what extent are the considered student variables likely to influence Mathematics Performance achievement. Third, school variables are added to model B as Level 2 explanatory variables (Model C). The final model (Model C) answers the question, which school variables contribute to explanatory effects on Mathematics Performance achievement, after controlling for student characteristics. By comparing Model B and Model C, a good understanding of how the student-level variables operate to influence Mathematics Performance achievement within schools of different characteristics can be estimated.

4. HLM Analysis

Ordinary least square (OLS) regression-based analysis that takes into consideration the data's hierarchical structure is known as hierarchical linear modelling (HLM). Hierarchical linear modelling is widely used for making data reductions and predictions. HLM is highly flexible model for analysing complex nested relationships as it allows specifying different relations across levels of the educational system (such as schools, classrooms, students, etc.). Data

that is hierarchically structured is nested data that is grouped into different clustered of units, such as students belong to classrooms at schools. Because the clusters of observations are not independent of each other, the nested structure of the data violates the independence requirement of OLS regression. Also, HLM approach determine the sources of the variance at each level of a hierarchical data structure linking repeated scores to students and students to classrooms, teachers, and schools. This allows understating the variances over the considered levels.

The unconditional model (Model A) is actually equivalent to a one-way ANOVA model with random effects:

Level 1:

Mathematics

achievement = $y = \beta_0 +$ Level 2:

$0 = 00 + 0$

where Mathematics Achievement, is the Mathematics performance achievement for student i in school j . The variance of ϵ_{ij} , the variability of random error at the student level, represents the variance of Mathematics Performance achievement between students within the school, and denoted σ^2 , and the variance of μ_j , denoted τ_{00} , is the variance of Mathematics Performance achievement between schools. The γ_{00} is the grand mean of Mathematics Performance achievement of all students. The estimates of μ_0 , σ^2 and τ_{00} are given in Table 3. We obtain a significant non-zero grand-mean mathematics achievement score, $\mu_0 = 394.3$ with $se = 3.53$. The level-1 variance estimate showed significant mathematics achievement score variation across students within a school, $\sigma^2 = 5711.6$. The level-2 variance shows a significant variance in the mathematics achievement means across schools, $\tau_{00} = 3128$. The Intra-class correlation (ICC; denoted as ρ) was calculated for the unconditional model to explore the relative school differences. Mathematically, the ICC is defined as $\tau_{00}/(\tau_{00} + \sigma^2)$. Therefore, the ICC in this study was 0.3539, meaning that the variation between schools accounted for 35% of the total variance of Mathematics Performance achievement. When we use

proc mixed with SAS, we obtain $\sigma^2 = 395.9$ with a $se = 4.51$. As expected, the coefficient estimates differ slightly but the standard error obtained by the macro is smaller as proc mixed assumes that schools are selected by a simple random sampling.

Now we add student level predictors to explain variation in mathematics achievement and obtain model B. We use the following model:

$$\text{Mathematics achievement}_{ij} = \mu_0 + \mu_{1j}(\text{BSBGHER}) + \mu_{2j}(\text{BSBGSLM}) +$$

$$\mu_{3j}(\text{BSBGEML}) + \mu_{4j}(\text{BSBGSCM}) + \mu_{5j}(\text{BSBG SVM}) + r_{ij} \text{ where}$$

$$\mu_0 = 00 + \sigma_0$$

$$\mu_{1j} = 10 + u_{1j}$$

$$\mu_{2j} = 20 + u_{2j}$$

$$\mu_{3j} = 30 + u_{3j}$$

$$\mu_{4j} = 40 + u_{4j}$$

$$\mu_{5j} = 50 + u_{5j}$$

$$\text{with } \text{var}(\mu_{1j}) = 11, \text{var}(\mu_{2j}) = 22, \text{var}(\mu_{3j}) = 33, \text{var}(\mu_{4j}) = 44, \text{var}(\mu_{5j}) = 55.$$

Each school mathematics achievement is identified by six parameters: the intercept μ_0 and the slopes $\mu_{1j}, \mu_{2j}, \dots, \mu_{5j}$. As predictors are centered around grand mean, the intercept represents the grand mean. These parameters vary across schools and all the estimated variance of the slopes are non-significant. We are not able to reject the hypothesis that $\sigma_0 = 0$, for $j = 1, 2, \dots, 5$. Hence the relationship between level 1 predictors and mathematics achievement within schools does not vary significantly across schools. While the fixed component for the intercept is still statistically significant, and its value (395.4) has changed very little. Also, all the five student-level variables are significant. Note that student confidence with mathematics was the strong predictor of mathematics achievement in Saudi Arabia; a one scale-point increase in confidence in mathematics increased achievement by 18.5 points. In a second position but far behind we found three predictors. For home educational resources, liking learning and value learning, a one scale-point increase increased mathematics score by less than 4 points. However, students who were interested in what teacher says or said

that teacher gives interesting things to do perform less than students who have less positive attitude; a one scale-point increase decreased mathematics score by more than 2 points.

Note that by comparing the σ^2 between model A and model B, we obtain the proportion reduction in variance or “variance explained” at level 1. Recall that Proportion of variance explained is $(\sigma^2 \text{ of model A} - \sigma^2 \text{ of model B}) / \sigma^2 \text{ of model A}$

The estimated proportion of variance between students within the school explained by the model with five student predictors is $(5711.6 - 4408.3) / 5711.6 = 0.23$

Which means, adding the five predictors of mathematics achievement reduced the within-school variance by 23%.

Now model C can be constructed by adding school level predictors. In this case, the mathematics achievement can be represented as

$$\text{Mathematics achievement}_{ij} = \mu_0 + \mu_{1j}(\text{BSBGHER}) + \mu_{2j}(\text{BSBGSLM}) +$$

$$\mu_{3j}(\text{BSBGEML}) + \mu_{4j}(\text{BSBGSCM}) + \mu_{5j}(\text{BSBG SVM}) + r_{ij},$$

$$\text{Where } \mu_0 = \mu_{00} + \mu_{01}\text{BCBGEAS} + \mu_{02}\text{BCBGDAS} + \mu_{03}\text{BCBGMRS} + \sigma_0, \text{ with}$$

$$\mu_{1j} = 10 + u_{1j} \quad \mu_{1j} = 10 + u_{1j}$$

$$\mu_{2j} = 20 + u_{2j}$$

$$\mu_{3j} = 30 + u_{3j}$$

$$\mu_{4j} = 40 + u_{4j}$$

$$\mu_{5j} = 50 + u_{5j}$$

The proportion reduction in variance or “variance explained” at level 2 is $(\sigma_{00}^2 - \sigma_{00}^2) / \sigma_{00}^2$.

The estimated proportion of variance between schools explained by the model with five student predictors is $(2273.6 - 1998.1) / 2273.6 = 0.12$. Hence 12% of the between-school variance in mathematics score is explained by the three level 2 predictors. After removing the effect of the five level 1 and the three level 2 variables, the correlation between scores in the same school is slightly reduced since $\sigma_{00}^2 / (\sigma_{00}^2 + \sigma^2) = 1998.1 / (1998.1 + 4355.6) = 0.31$. Table 3 presents parameter estimates of student and school levels of the three models A, B, and C.

Table 4. Parameter estimate of models A, B, and C.

Variable	Model A		Model B		Model C	
	Estimate	SE	Estimate	SE	Estimate	SE
Student Level						
INTERCPT ₀₀	394.3***	3.53	395.4***	3.06	395.5***	2.95
Home educational resources			3.8***	0.77	3.7***	0.76
Student like learning mathematics			3.8***	0.87	4.0***	0.88
Student ENGAGED with mathematics			-2.5*	1.01	-2.7**	1.01
Student value learning mathematics			-3.6***	0.87	-3.8***	0.90
Student CONFIDENCE with mathematics			18.5***	0.86	18.4***	0.88
School Level						
School discipline and safety					-1.4	1.19
School emphasis on academic success					7.9***	1.44
Instruction affected by mathematics resources shortages					-4.0*	1.84
Level 1 variance ₂	5711.6		4408.3		4355.6	
Level 2 variance ₀₀	3128.0		2273.6		1998.1	

* $p < .05$. ** $p < .01$. *** $p < .001$.

Based on the findings in Table 4, the associations between the student-level variables and Mathematics Performance achievement in Model C are quite similar to those in Model B. All the five level 1 variables are significant. The mathematics achievement still strongly affects by the student confidence in mathematics. There is no statistically significant relation between school discipline and mathematics achievement. The remaining two variables, identified as the school-level variables, have a significant impact on score. However, mathematics resources shortages are found to have a negative influence on Mathematics Performance achievement.

5. Conclusions

In this chapter, the hierarchical linear modelling technique is used to explain causally the Mathematics Performance achievement of students in Saudi Arabia. We found that 35% of the variance in students' Mathematics Achievement is due to the variance of performance between schools and this also corresponds to the intra-class correlation. The results of PISA 2006 show that the

intra-class correlation ranges from 0.06 in Finland to 0.61 in Hungary. This ranks Saudi Arabia in the medium position for the variability in performance linked to differences between schools. All the student level factors are found to be significant but their impact on achievement do not vary significantly across the population of schools. At the school level, it is found that School discipline and safety and instruction affected by mathematics resources shortage have a significant impact on performance. However, a scale point increase in the availability of school resources for mathematics decreased achievement by 4 points.

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Data Availability Statement: TIMSS 2011 data set which is available at <https://timssandpirls.bc.edu/timss2011/international-database.html>

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Akyüz, G. "The effects of student and school factors on mathematics achievement in TIMSS 2011, 2014, 39(172).
2. Alkhateeb, H.M. Gender differences in mathematics achievement among high school students in the United Arab Emirates, 1991-2000. *School Science and Mathematics*, **2001a**, 101(1), 5 – 9.
3. Alkhateeb, H.M. Gender differences in mathematics achievement among high school students in the United Arab Emirates, 1991-2000. *School Science and Mathematics*, **2001b**, 101(1), 5 – 9.
4. Anderman, E.M., & Kimweli, D.M.S. School violence during early adolescence. *Journal of Early Adolescent*, **1997**, 17 (4), 408-438.
5. Borman, G., & Dowling, M. Schools and inequality: A multilevel analysis of Coleman's equality of educational opportunity data. *Teachers College Record*, **2010**, 112 (5), 1201-1246
6. Chiu, M. M. Families, cultures, and science achievement in 41 countries: Country, schools, and student-level analyses. *Journal of Family Psychology*, **2007**, 21(3), 510
7. Coleman, J. S., Campbell, E. Q., Hobson, C. J., McPartland, F., & Mood, A. M. i Weinfeld, F.D. Equality of educational opportunity, **1966**.
8. Else-Quest, N. M., Hyde, J. S. & Linn, M. C. Cross-National Patterns of Gender Differences in Mathematics: Meta-Analysis. *Psychological Bulletin*, **2010**, 136(1), 103–127.
9. Gibbs, R., & Poskitt J. Student Engagement In the Middle Years of Schooling (Years 7 – 10): A Literature Review. Report to the Ministry of Education, **2010**.
10. Gorard, S., & See, B. H. The impact of socio-economic status on participation and attainment in science. *Studies in Science Education*, **2009**, 45(1), 93-129.
11. Goddard, R. D., Sweetland, S. R., & Hoy, W. K. Academic emphasis of urban elementary schools and student achievement: A multi-level analysis. *Educational Administration Quarterly*, **2000**, 36, 683-702.
12. Gorard, S., & See, B. H. The impact of socio - economic status on participation and attainment in science. *Studies in Science Education*, **2009**, 45 (1), 93-129.
13. Guiso, L., Monte, F., Sapienza, P., & Zingales, L. Culture, gender, and math. *Science*, **2008**, 320, 1164–1165.
14. Hanushek, E. A. Assessing the effects of school resources on student performance: An update. *Educational Evaluation and Policy Analysis*, **1997**, 19(2), 141–164.
15. Hedges, L.V., Laine R. D. and Greenwald, R. Does Money Matter? A Meta-analysis of Studies of the Effects of Differential School Inputs on Student Outcomes. *Educational Researcher*, **1994**, 23(3), 5– 14.
16. House J. D., & Telese J. A. Relationships between student and instructional factors and algebra achievement of students in the United States and Japan: an analysis of TIMSS 2003. *Educational Research and Evaluation*, **2008**, 14(1), 101 –112.
17. Hoy, W. K. School characteristics that make a difference for the achievement of all students: A 40-year academic odyssey. *Journal of Educational Administration*, **2012**, 50, 76-97.
18. Hoy, W. K., Tarter, C. J., & Kottkamp, R. B. Open schools/healthy schools: Measuring organizational climate. Beverly Hills, CA: Sage, **1991**.
19. Hyde, J. S., Fennema, E., & Lamon, S. J. Gender differences in mathematics performance: a meta-analysis . *Psychological bulletin*, **1990**, 107(2), 139
20. Jurdek, M. Toward equity in quality in mathematics education: Springer, **2009**.

21. Kim, J. A., Namgung, J. Y. and Kang, S. J. The Effects of Background Variables on Math Achievements in 15 Years Old Korea Students. *Journal of Korea Society of Mathematics Education: The Mathematical Education*, **2004**, 43, 187–97.
22. Ma, X., & Klinger, D. A. Hierarchical linear modeling of student and school effects on academic achievement. *Canadian Journal of Education*, **2000**, 25, 41 – 55.
23. Martin, M. O., & Mullis, I. V. S. & Foy, P. TIMSS 2007 International Science Report Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades. TIMSS & PIRLS International Study Center Lynch School of Education, Boston College, Boston, **2008**.
24. Martin, M. O., Mullis, I. V., Gregory, K. D., Hoyle, C., & Shen, C. Effective schools in science and mathematics. IEA's third international mathematics and science study, IEA: Chestnut Hill, MA, **2000**.
25. Ministry of Education. Mathematics syllabus (primary). Singapore: Curriculum Planning & Development Division, **2000**.
26. Mullis, I. V., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. TIMSS 2003 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades. TIMSS & PIRLS International Study Center. Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467, **2004**.
27. Papanastasiou, C. School, effects of attitudes and beliefs on mathematics achievement. *Studies in Educational Evaluation*, **2000**, 26, 27 – 42.
28. Patrick, H., Ryan, A., & Kaplan, A. Early adolescents' perceptions of the classroom social environments, motivational beliefs, and engagement. *Journal of Educational Psychology*, **2007**, 99(1), 83- 98.
29. Prothrow-Stith, D., & Quaday, S. Hidden casualties: The relationship between violence and learning. Washington, DC: National Health & Education Consortium and National Consortium for African American Children, Inc, **1995**.
30. Raudenbush, S.W., A. S. Bryk. Hierarchical linear models: Applications and data analysis methods. Thousand Oaks: Sage Publications, **2002**.
31. Riegle-Crumb, C. The cross-national context of the gender gap in math and science. In L. Hedges & B. Schneider (Eds.), *The social organization of schooling* (pp. 227–243). New York, NY: Russell Sage Foundation, **2005**.
32. Senler, B., & Sungur, S. Parental influences on students' self - concept, task value beliefs, and achievement in science *The Spanish journal of psychology*, **2009**, 12 (1), 106 117.
33. Sun, L., Bradley, K. D., & Akers, K. A multilevel modelling approach to investigating factors impacting science achievement for secondary school students: PISA Hong Kong sample. *International Journal of Science Education*, **2012**, 34(14), 2107–2125.
34. Shernoff, D., & Schmidt, J. Further evidence of an engagement-achievement paradox among US high school students. *Journal of Youth and Adolescence*, **2008**, 37, 564-580.
35. Tomul, E., & Celik, K. The relationship between the students' academics achievement and their socioeconomic level: cross regional comparison. In H. Uzunboylu, & N. Cavus (Eds.), *World Conference on Educational Sciences-New trends and issues*, **2009**.
36. Tsao, Y. L. A comparison of American and Taiwanese students: Their math perception. *Journal of Instructional Psychology*, **2004**, 31, 206 – 213.
37. Wang, J. A trend study of self-concept and mathematics achievement in a cross-cultural context, *Mathematics Education Research Journal*, **2007**, 19 (3), 33 – 47.
38. Wang, J., & Wildman, L. An empirical examination of the effects of family commitment in education on student achievement in seventh grade science, *Journal of Research*

in Science Teaching, **1995**, 32 (8) ,
833-837.