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On Fuzzy Sine Chaotic Based Model in Security Communications

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Abstract

In the fuzzy chaos based dynamical model a sine chaotic map is proposed, where the model works through designing the Takagi-Sugeno fuzzy model with If-Then fuzzy rules combined with discrete-time chaotic map as a seed map in continuous map. The chaotification method was applied through using a sine chaotic map and approached on the outputs of the Takagi-Sugeno fuzzy model. That applies the model on all chaotic trajectories of the premise variable. The proposed model requires the application on a two dimensional Lozi chaotic map as a discrete-time chaotic map. Appropriate parameters proposed to ensure the chaotic behavior for the Lozi map. Numerical simulations from Lozi chaotic outputs by supposed fuzzy model, and presented after many iterations to compute the trajectory with sine map to be in closed interval [-1,1].Absolute values on a sine chaotic map ensure positive values in [0,1]. The model inherits features of chaotic maps. The results demonstrate the effectiveness of the model with initial values equal to zero, and the model could be implemented in generating the secret key in cryptosystems and secure communication.

Keywords: Fuzzy chaotic, Sine chaotic map, Takagi Sugeno fuzzy model, Lozi map.

1. INTRODUCTION

A Fuzzy Logic that combines Chaotic Systems [1] as a Fuzzy Modeling of Chaotic Systems[2], for both discrete and continuous time models. Fuzzy logic systems lack a systematic modeling and control design methodology if used individually, and in most controllers the stability of the closed-loop controlled system is not easy. So a fuzzy model for chaotic dynamics was approached in many applications [2]. The sine map is employed to compose chaotic maps to get Sine Chaotic Map SCM based on Cosine Chaotic Map CCM as in reference [3]. The SCM is in the interval [-1,1] for any value for the chaotic map that was used as seed map and with zero value as it started with initial zero value. That SCM determines the outputs within the sine outputs interval.

In this paper, the Takagi-Sugeno TS fuzzy model [4] is applied by a chaotic system, where the discrete-time model is used for the performance. The proposed method is efficient in that it inherits the use of TS fuzzy model in many problems. The TS fuzzy model determines the trajectory of the premise variable in the chaotic system's region. The sine map is implemented on a chaotic map (as seed map), that is sine for all chaotic system trajectory, and then avoiding the negative values for sine map. A discrete time chaotic map uses the Lozi map with appropriate parameters that ensure the chaotic behavior for trajectory values. The proposed method can provide a perfect combination between the TS chaotic fuzzy model with sine chaotic map. However, our proposed method could be implemented on any discrete-time chaotic map.

The organization of this paper is as follows. In Section 2, we shall give a mathematical approach for chaotification method sine chaotic map, and its terminology. Section 3, presents steps of suggesting the TS fuzzy model for chaotic systems. In Section 4, we shall study and implement a simulation example which illustrates the effective performance of the proposed model. Finally, in section 5, we will give the conclusion of the paper.

2. Sine Chaotic Map:

The SCM is a new chaotification method that enhances the chaotic complexity of existing chaotic maps. This method perform the sine function alongside a proposed chaotic map or the original maps or seed maps that cascade in used system, the way of performing is similar to the cosine function. So the result(s) pay a new chaotic maps have a wide chaotic range within the closed interval [-1,1] such; elevated sensitivity, complex characteristics, high nonlinearity, and an extended cycle length in compared with the seed map.

Theoretically SCM has properties based on the properties of the underlying seed maps. It take any chaotic maps(continuous or discrete) as seed map and improve its chaotic complexity, or chaotic parameter(s) range. So it will employed on Lure type discrete-time systems in [1] .

Sine Chaotic Maps Terminologies :

The sine function is a trigonometric function, which is a real valued function, periodic, and produce waves. Also this performance leverage the sine function (wave like) properties to enhance the chaotic behavior for different chaotic maps. The resulting SCM is bounded map in [-1,1], and it depict the chaotic behavior instead of periodic patterns.

A SCM can be directly formulated as cascade system with form: $x_{n+1} = C(F(x_n))$

Where $C(x) = Sin(x)$ is the sine function, and F is the chaotic map as seed map instead of cosine map in [5].

From properties of sine function the largest difference between the outputs is equal to 2 which is the bounds on the outputs interval, since $(1-(-1)) = 2$.

So from trying to increase chaotic complexity it must use an existing chaotic map instead of linear functions. That interpreted a new control parameter(s) to gain large differences between seed values for seed map. Since from the chaos map features is the sensitivity to initial conditions [6]. The sensitivity refer to that small differences pay a large differences [7]. So with new control parameter s the SCM could be formulated as;

$$x_{n+1} = C\left(2^{(s+F(x_n))}\right)$$

where s is chosen within interval $[10,25]$, if $s < 10$ the result will be in a periodic state

$$x_{n+1} = C\left(2^{(9+F(x_n))}\right) = C\left(2^{(9)}.2^{(F(x_n))}\right)$$

if $s < 25$ it will bounds the computational complexity of the resulting system

$$x_{n+1} = C\left(2^{(30+F(x_n))}\right) = C\left(2^{(30)}.2^{(F(x_n))}\right)$$

Since whatever the angle is small or large a small difference in $F(x)$ led to wide diverge in output(s) from the accumulation error .

Suggest TS Fuzzy Model for Chaotic System

There are two types for the process of TS fuzzy modeling of chaotic systems, the continuous-time and discrete-time TS fuzzy model [1]. We will work on the discrete-time type with two dimension. The idea is to use the discrete-time model to get a trajectory for point(s) in phase space of chaotic system for a prime variable. Details of the design and the algorithm for the fuzzy chaotic cryptosystem in explained in the steps below:

Step 1: Suppose a 2-dimensional discrete-time chaotic system $x(t) = [x_1(t),\ x_2(t)]$.

Step 2: Determine which the primitive variables in the system $x_1(t),\ x_2(t)$

Step 3: Design the TS fuzzy model through if-then rules on values of primitive variable at the chaotic system such; $x_1(t)$ on the interval $[-d, d]$.

Step 4: Calculate the table values for iterations of the TS system implemented on initial values $x_1(0), x_2(0)$ to find the phase trajectory.

Step 5: Perform the sine function alongside a chaotic map (seed map).

Step 6: Formulate the resulted map sine chaotic system on $x(t) = [x_1(t),\ x_2(t)]$.

Step 7: Evaluate the parameters in the system and the values of variables to get the system matrix A_i.

Step 8: Determine the output values that represent the trajectory for $x_1(t)$ in the phase trajectory after $t = T$ iteration .

Consider a class of discrete-time nonlinear control system without input term constructed as;

$$x(t+1) = f(x(t))$$

where $x(t) \in R^2$, $[x_1(t) \;\; x_2(t)]^T$ is the state vector and $f(x(t)) \in R^2$ is a nonlinear function with appropriate dimension defined on $x(t)$. Where $(t+1)$ is index of time steps in discrete case of the system. The TS fuzzy model here is composed for the rules as a set of rules (fuzzy implications) :

$\boldsymbol{Plant\ Rule\ i}$: $IF\ x_1(t)\ is\ \Gamma_1^i\ and\ x_2(t)\ is\ \Gamma_2^i$
$THEN\ x(t+1) = A_i x(t) + b_i(t)$

For $i = 1, 2, \ldots, q$. $x_1(t), x_2(t)$ are the premise variables which consist of the state values in states space for the system, q is the number of rules of this TS fuzzy model, Γ_j^i are fuzzy sets for $x_j(t)$ and $j = 1,2$. $A_i \in R^{2\times 2}$ is a discrete-time control system matrix and $b_i(t) \in R$ denotes the bias terms.

These rules characterize local relation(s) of the chosen system in the state space. Mainly, the essential feature for TS model is expressing the local dynamics of each fuzzy implication through a linear state-space system model. The fuzzy system is then modeled through fuzzy blending of the local linear system models by some appropriate membership functions.

3. Generate the TS Fuzzy Chaotic-Based Model.

The input to the system is $x(t)$ initial values for vector states from the vector variable $x(t) = [x_1(t) \quad x_2(t)]^T$ namely $x_1(t)$ to be the output $[x_1(t-0) \quad x_1(t-1) \quad \cdots \quad x_1(t-j+1)$.
And compute;

$$sin(x_1(t)) = [sin\,(x_1(t-0)) \quad sin(x_1(t-1)) \quad \cdots \quad sin(x_1(t-j+1))]$$

.

Now, a through the form the absolute value ensure all the terms in the trajectory are positive.

4. Simulation Example on the Suggested Fuzzy Sine Chaotic Model

In this simulation we will consider the chaotic map of Rene Lozi (known as Lozi map) [8] in a chaotic cryptosystem. That is a discrete-time chaotic Lozi system in view of step by step of performing the suggested system. Its chaotic behavior exhibited in a single scroll, since it is well-known two-dimensional map on the interval [0,1], the Lozi map as implicational example for theoretical derivative is like the Hénon map but with absolute formulate the dynamical equations as the equation system bellow [9]:

$$x_1(t+1) = -1.8\,|x_1(t)| + x_2(t) + 3$$
$$x_2(t+1) = 0.25 x_1(t)$$

The Lozi Map is with only one variable in the nonlinear term [12], which has the nonlinear term$|x_1(t)|$. Since$|x_1(t)|$ is not well defined at $x_1(t) = 0$, let $\varphi(x_1(t) = |x_1(t)|$ and choose $x_1(t)$ to be the premise variable. The equivalent fuzzy model can be constructed with System matrices as:

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix}$$

the system matrices evaluated to be equaled, and the non-common bias terms as;

$$b_1 = \begin{bmatrix} 3 - 1.8d \\ 0 \end{bmatrix},\ b_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

The fuzzy sets for prime variable values are determined through the membership functions as;

$$F_1(x_1(t)) = (|x_1(t)|/d)$$
$$F_2(x_1(t)) = 1 - (|x_1(t)|/d)$$

With $d = 3.5$, so $x_1(t) \in [-3.5, 3.5]$
Then general TS-fuzzy model of discrete time chaotic system at Lozi map with dynamical equations can be written as follows ;

$\boldsymbol{Rule\ i}$: $IF\ x_1(t) is\ \mathit{\Gamma}_i$
$THEN\ x(t+1) = A_i x(t) + bi(t)$, For $i = 1,2, \ldots, r$

where the premise variable $x_1(t)$ is a proper state variable, and $\mathit{\Gamma}_1, \mathit{\Gamma}_2$ are triangular fuzzy sets "about -3.5",and "about 3.5", respectively. The model infer the rules $r = 2$;

$\boldsymbol{Rule\ 1}$: $IF\ x_1(t) is\ \mathit{\Gamma}_1$
$THEN\ x(t+1) = A_1 x(t) + b_1(t)$

And $\boldsymbol{Rule\ 2}$: $IF\ x_1(t) is\ \mathit{\Gamma}_2$
$THEN\ x(t+1) = A_2 x(t) + b_2(t)$

So
$\boldsymbol{Rule\ 1}$: $IF\ x_1(t) is\ "about - 3.5"$

$$THEN\ x(t+1) = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} x(t) + \begin{bmatrix} -3.3 \\ 0 \end{bmatrix}$$

***Rule* 2**: $IF\ x_1(t) is$ "$about$ 3.5"

$$THEN\ x(t+1) = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

the model is with tiny difference in values at bias terms. From the observation of bias terms, the systems Lozi map has non-common bias terms, while there are other systems have common bias terms in fuzzy models. Since $\varphi(x) = \sum_{i=1}^{r} \mu_i d_i$, Here $\varphi(x) = 1$

4.1 The Sine Lozi Chaotic SLC Map :

The model performed on Lozi map as well known system to be exact represented by TS fuzzy models on the vector variables $x(t) = [x_1(t) \ \ x_2(t)]^T$, where $x(t) \in R^2$, $x_1(t)$, $x_2(t)$ is the state vector.

These features will help in encrypting the message in this work through composing the increasing sequence μ_n with the super increasing sequence S

$$\begin{cases} x_1(t+1) = 3 - a|x_1(t)| + x_2(t) \\ \qquad x_2(t+1) = \beta\, x_1(t) \end{cases}$$

Where α, β are the parameters of the system, such that $\alpha \in [1, 1.8]$, $\beta \in [0, 0.4]$, if we take some values for these parameters to view the chaotic behavior the dynamical system be as;

$$\begin{cases} x_1(t+1) = 3 - 1.8|x_1(t)| + x_2(t) \\ \qquad x_2(t+1) = 0.25\, x_1(t) \end{cases}$$

The fuzzy sets that defined on values of premise variable $x_1(t)$ are as;

$$F_1(x_1) = |x_1|/d\ , \ \ F_2(x_1) = 1 - |x_1|/d$$

For $d \in R^+$, suppose $d = 3.5$, that is $x_1(t) \in [-d, d] = [-3.5, 3.5]$, and $\sum_{i=1}^{2} F_i = 1$

The system matrices that associated with the equations system are;

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ 0.25 & 0 \end{bmatrix}$$

Since there is no nonlinear terms in the system of equations, and so the bias terms b on two rules will be as;

$$b_1 = \begin{bmatrix} 3 - 1.8d \\ 0 \end{bmatrix} \ \ b_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

To control on $x_1(t)$ values. Since the system may be seen as;

$$\begin{cases} x_1(t+1) = 3 - 1.8|x_1(t)| + 0\, x_1(t) + x_2(t) \\ x_2(t+1) = 0 \qquad\qquad + 0.25\, x_1(t) + x_2(t) \end{cases}$$

$x(t)$, $b_i \in R^2$, $A_i \in R^{2\times 2}$, $for\ i = 1, 2$. In designing the sine Lozi map(CLZ)map the system be;

$$f\big(x_n(t+1)\big) = C(2^{(s+F(x_n(t)))})$$

So

$$\begin{cases} x_1(t+1) = \sin\,(2^{(s+3-1.8|x_1(t)|+x_2(t))}) \\ \qquad x_2(t+1) = \sin(\,2^{(s+0.25\,x_1(t))}) \end{cases}$$

Where $C(x) = sin(x)$, and $s \in [10, 25]$. So then $x_1(t+1)$, $x_2(t+1) \in [-1, 1]$, for initial conditions let the values $x(0) = [x_1(0) \ \ x_2(0)]^T = [1 \ \ \ 0.25]$,

As experiment : The system performed with initial values for premise variables as; $x_1(0) = 1$, $x_2(0.25)$, and $s = 10$ after 40 iterations. Then compute the absolute value for the trajectory of $x_1(t)$ to avoid negative values for sine map.

TABLE 1: *Sine map values on TS fuzzy chaotic model values for initial values* $x_1(0) = 1$, $x_2(0.25)$, *and* $s = 10$

t. t	$x_1(t)$	$x_2(t)$	*sin(2power x1)*	*| sin x|*	*sin(2power x2)*
0	1	0.25	-0.313057013	0.313057013	-0.92831835
1	1.45	0.3625	0.998066961	0.998066961	-0.179182443
2	0.7525	0.188125	-0.397093223	0.397093223	-0.888460467
3	1.833625	0.45840625	-0.622320818	0.622320818	-0.42037907
4	0.15788125	0.039470313	-0.898656896	0.898656896	0.031437117
5	2.755284063	0.688821016	0.678144995	0.678144995	-0.967890576
6	-1.270690297	-0.317672574	-0.289285932	0.289285932	-0.995633513
7	0.395084891	0.098771223	0.917849586	0.917849586	-0.145308166
8	2.387618418	0.596904605	-0.861776073	0.861776073	0.037952507
9	-0.700808548	-0.175202137	0.994655	0.994655	0.852846436
10	1.563342476	0.390835619	-0.815126054	0.815126054	-0.916887848

11	0.576819163	0.144204791	0.514472534	0.514472534	0.62169329
12	2.105930298	0.526482575	-0.400518843	0.400518843	-0.99998642
13	-0.264191962	-0.066047991	-0.957244315	0.957244315	-0.909413844
14	2.458406478	0.614601619	-0.986620123	0.986620123	-0.225684679
15	-0.810530041	-0.20263251	-0.461563713	0.461563713	-0.680105406
16	1.338413417	0.334603354	0.680110472	0.680110472	-0.100645571
17	0.925459204	0.231364801	-0.223844926	0.223844926	0.895841474
18	1.565538233	0.391384558	0.659943601	0.659943601	-0.994966707
19	0.573415738	0.143353935	-0.082994123	0.082994123	0.003691331
20	2.111205606	0.527801401	0.752602264	0.752602264	-0.214935454
21	-0.272368689	-0.068092172	-0.389376691	0.389376691	0.24075505
22	2.441644188	0.610411047	0.698242097	0.698242097	-0.923978473
23	-0.784548491	-0.196137123	-0.648345379	0.648345379	0.998712809
24	1.391675594	0.347918898	-0.676282801	0.676282801	0.472568703
25	0.842902829	0.210725707	0.903755763	0.903755763	-0.616447945
26	1.693500614	0.423375154	0.712247936	0.712247936	-0.362316663
27	0.375074048	0.093768512	0.759033837	0.759033837	-0.486825173
28	2.418635226	0.604658807	0.735526156	0.735526156	-0.898354744
29	-0.748884601	-0.18722115	-0.123689894	0.123689894	0.770372491
30	1.464786569	0.366196642	-0.820122736	0.820122736	0.404285776
31	0.729580818	0.182395205	0.996973621	0.996973621	-0.378889699
32	1.869149732	0.467287433	0.709360782	0.709360782	0.920939027
33	0.102817916	0.025704479	0.084254622	0.084254622	-0.565190935
34	2.84063223	0.710158057	0.351946819	0.351946819	-0.701896588
35	-1.402979956	-0.350744989	-0.722139753	0.722139753	-0.948468961
36	0.123891089	0.030972772	-0.528245544	0.528245544	-0.071205426
37	2.807968812	0.701992203	0.93393977	0.93393977	0.679799012
38	-1.352351658	-0.338087915	-0.877713171	0.877713171	-0.440067509
39	0.227679101	0.056919775	-0.860323267	0.860323267	-0.206824369
40	2.647097394	0.661774348	-0.672508969	0.672508969	-0.87528235

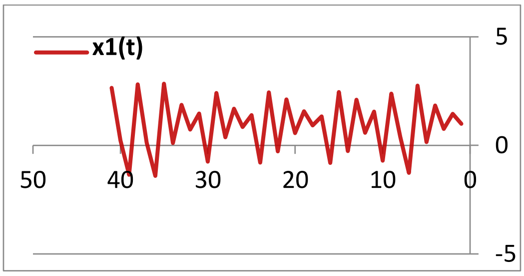

FIGURE 1: *Values for $x_1(t)$ for iteration 40, as in the table (1) .*

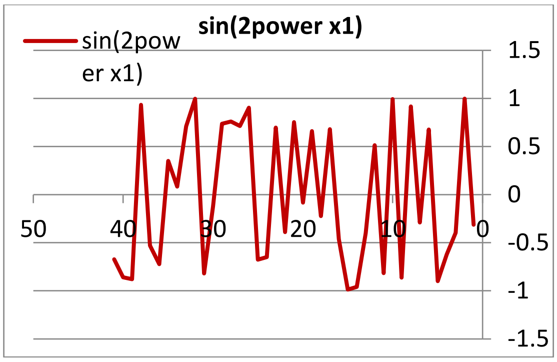

FIGURE 2: *Sine Chaotic values for $x_1(t)$ with iteration 40, as in the table (1)* .

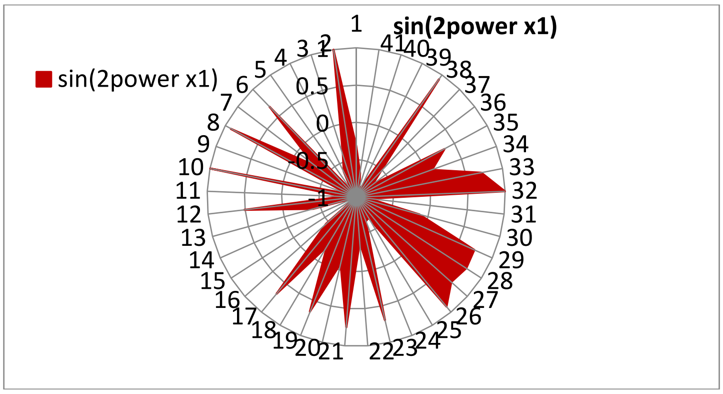

FIGURE 3: *Sine Chaotic values for $x_1(t)$ with iteration 40 as area*

5. Conclusions:

In this work, a designing of fuzzy model-based for chaotic map has been proposed. It performed in combination with the TS fuzzy model for discrete-time chaotic systems, that were exactly derived with only one premise variable. Following the sine map implemented on the chaotic map at Lozi chaotic map, with non-common bias terms and the same premise variable and driving signal. In this fuzzy chaos model-based, a sine chaotic map were applied with numerical data for chaotic behavior, parameters data and premise variable with initial values in interval [-3.5,3.5]. The advantage of this performance design is that all well-known chaotic systems stated achieved the results, since the trajectory values for premise variable, converted into values for sine map in interval [-1,1]. Numerical simulations with the values figures are shown to be consistent with theoretical design. A good result were gained and the security of the system shown in flexibility in changing the supposed values that will make the system behavior unpredictable. Assume initial values randomly from closed interval in state space; Choosing the chaotic map is unpredictable. Finding the trajectory (orbit) of points, with increase iterations for absolute value for sine map for the seed Lozi chaotic map.

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