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Traits of Nano regular weakly neighbourhood and Nano regular weakly closed map in Nano Topological space

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Abstract

The aim of this paper is to introduce new forms of neighbourhood called Nano regular weakly neighbourhood. In addition, we define Nano regular weakly closed maps also various properties and characterizations related to these functions are also examined and investigate some of their properties.

Keywords: Nrw-neighbourhood and Nrw-closed map.

1. INTRODUCTION

In the past few decades, there has been a significant increase in the applications of Topology in fields as diverse as Medicine, Engineering, Economics, Chemistry, Computer Science, Cosmology. Topology has gained access to human interventions to make life easier. In recent years, mathematicians use topology to model and give solutions to the real world problems. Many different versions of generalized closed maps and related topological properties were introduced by eminent Mathematicians.

Stone [1] and Tong [2] defined and investigated Regular open sets and strong regular open sets respectively. Cameron [3] introduced and analyzed the weaker form of regular open sets called a regular semiopen set. Benchalli et. al., [4] defined and studied RW-closed sets.

Many different versions of generalized closed maps and related topological properties were introduced by eminent Mathematicians. Long and Herrington [5] studied the properties of regular closed functions. Malghan [6] were introduced and studied Generalized closed mappings. The notion of generalized pre-closed functions and semi regular weakly closed maps were introduced and studied by Noiri et., al., [7] and Wali et. al., [8] respectively.

Palwak [9] defined and discussed the notion of rough sets. The idea of nano topological space was initiated by Lelli's Thivagar [10] in terms of lower, upper approximations and its boundary regions. He applied set valued ordered information system for attribute reduction in nutrition modelling. The nano α-open sets, nano semi-open sets, nano pre-open sets and nano regular-open sets are also studied in [10]. Dayana Mary S et.al. [11] introduced and analyzed nano regular weakly closed set in nano topological spaces.

P. Sathishmohan et.al [12] introduced and investigated the properties of nano semi pre-neighbourhoods. K. Chitrakala et., al., [13] defined and discussed Some generalization of neighbourhoods in nano topological spaces.

Lelli's Thivagar [14] introduced and studied a Nano closed map in Nano Topological Space. M. Bhuvaneswari and N. Nagaveni [15] defined and investigated Nwg-closed map in Nano

Topological Space. Hula M. Salih [16] presented Nano (θG) closed map, and studied the relationship between other current maps in nano topological spaces.

Moreover, in this paper we introduce the concept of Nano regular weakly neighbourhood and study their basic properties. Here, we also define Nano regular weakly closed maps and investigate several properties of these functions..

Throughout this paper, $\left(P, \tau_R\,(S)\right)$ is a Nano Topological space with respect to S where $S \subseteq P$,R is an equivalence relation on P , P/R denotes the family of equivalence classes of P by R , on which no separation axioms are assumed unless otherwise mentioned. $\left(Q, \tau_R\,(T)\right)$ is a Nano Topological space with respect to T, where $T \subseteq Q$, R is an equivalence relation on Q, Q/R denotes the family of equivalence classes of Q by R . $(W, \tau_R(X))$ is a Nano Topological space with respect to Z, where $Z \subseteq W$, $R$is an equivalence relation on W , W/R denotes the family of equivalence classes of W by R. For a subset M of a space $\left(P, \tau_R\,(S)\right)$, $Ncl(M)$ and $Nint(M)$ denote the nano closure of M and the nano interior of M respectively. We provide the definitions of some of them which are used in our present study.

2. Preliminaries

Throughout this paper $\left(P, \tau_R(S)\right)$ (or simply P) always mean a Nano topological space on which no separation axioms are expected unless generally specified.

This section is to recall some definitions and properties which are very useful in the subsequent sections.

Definition 2.1 [9] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. The pair *(U, R)* is said to be the approximation space. Let $X \subseteq U$. Then,

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
$L_R(X) = \bigcup_{x \in U} \{R(x): R(x) \subseteq X\}$, where *R(x)* denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. $U_R(X) = \bigcup_{x \in U} \{R(x): R(x) \cap X \neq \emptyset, x \in U\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to X and it is denoted by $B_R(X)$.
$B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [9] If (U, R) is an approximation space and$X, Y \subseteq U$, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.
(ii) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$.
(iii) $L_R(U) = U_R(U) = U$.
(iv) $U_R(X \cup Y) = U_R(X) \cup\ U_R(Y)$.
(v) $U_R(X \cap Y) \subseteq U_R(X) \cap\ U_R(Y)$.
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.
(vii) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
(viii) $L_R(X) \subseteq\ L_R(Y) and U_R(X) \subseteq U_R(Y) whenever X \subseteq Y$.
(ix) $U_R(X^c) =\ [L_R(X)]^c and L_R(X^c) = [U_R(X)]^c$.
(x) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$.
(xi) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 2.3 [10] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and $\emptyset \in \tau_R(X)$.
(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano Topology on U with respect to X. Then $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano Topology are known as Nano open sets in *U*. Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano Topology of $\tau_R(X)$.

Definition 2.4 [10] If τ_R is the Nano topology on *U* with respect to *X*, then the set β_R={U,$\emptyset$, $L_R(X), B_R(X)$} is the basis for $\tau_R(X)$.

Definition 2.5 [10] If $(U, \tau_R(X))$ is a Nano Topological space with respect to X and $X \subseteq U$and if $A \subseteq U$, then

(i) The Nano interior of the set *A* is defined as the union of all Nano open subsets contained in *A* and is denoted by *Nint(A)*. *Nint(A)* is the largest Nano open subset of *A*.

(ii) The Nano closure of the set *A* is defined as the intersection of all Nano closed sets containing *A* and is denoted by *Ncl(A*). *Ncl(A)* is the smallest Nano closed set containing *A*.

Definition 2.6 [10] If $(U, \tau_R(X))$ is a Nano Topological space with respect to *X*with $A \subseteq U$. Then *A* is said to be

(i) Nano-semi open, if $A \subseteq Ncl\big(Nint(A)\big)$.

(ii) Nano-regular open, if $A \subseteq Nint\big(Ncl(A)\big)$.

Definition 2.7 [11] A subset *A* of a Nano Topological space $(U, \tau_R(X))$ is called nano regular semi-open, if there is a nano regular open set such that $P \subseteq A \subseteq Ncl(P)$.

Definition 2.8 [11] A subset A of a Nano topological space $(U, \tau_R(X))$ is called nano semi regular open, $A = Nsint[Nscl(A)]$.

Definition 2.9 [11] Let $(U, \tau_R(X))$ be a Nano Topological space. A subset *A* of $(U, \tau_R(X))$ is called Nrw-closed, if $Ncl(A) \subseteq V$, where $A \subseteq V$ and *V* is Nano regular semi-open in $(U, \tau_R(X))$.

Corollary 2.10 [11] Every Nano closed set is Nrw-closed.

Definition 2.12 [14] A function$f: (U, \tau_R(X)) \to (V, \tau_R(Y))$ is a Nano-open map if the image of every Nano-open set in *U* is Nano-open in in *V*. The mapping *f* is said to be a nano-closed map if the image of every Nano-closed set in *U* is Nano-closed in in *V*.

3. NANO REGULAR WEAKLY NEIGHBOURHOOD

Definition 3.1

Let *b* be a point of $\big(U, \tau_R(X)\big)$ and *M* be a subset of *U*. Then *M* is called Nrw-neighbourhood of *b* in $\big(U, \tau_R(X)\big)$, if there exist a Nrw – open set *C* of $\big(U, \tau_R(X)\big)$ such that $b \in C \subseteq M$.

Theorem 3.2 Let *M* be a subset of $\big(U, \tau_R(X)\big)$. Then $b \in$ *Nrwcl(M)* if and only if for any Nrw-neighbourhood N_b of *b* in $\big(U, \tau_R(X)\big), M \cap N_b \neq \emptyset$.

Proof Suppose N_b is such that $M \cap N_b = \emptyset$------(1) we get $M \cap Y = \emptyset$, which implies that *M*is contained in the complement of *Y* (say Y^C). Then *Nrwcl(M)*$\subseteq Y^C$, because Y^C is a Nrw-closed set containing i. Hence *b* does not belong to *Nrwcl(M)*, which is a contradiction to the hypothesis. Therefore, our assumption (1) is wrong and hence $M \cap N_b = \emptyset$, where N_b of *b* is Nrw-neighbourhood in$\big(U, \tau_R(X)\big)$.

Conversely let us consider, for each Nrw-neighbourhood$N_b$such that $M \cap N_b \neq \emptyset$. To prove $b \in$ *Nrwcl(M)*. Suppose$b \notin$*Nrwcl(M)* then there exists a Nrw-closed set *Z* of $\big(U, \tau_R(X)\big)$, such that $M \subseteq Z$ and *b* does not belong to *Z*. Hence *b* belongs to the component of *Z* (say Z^C) and also the component of *Z* is Nrw-open set in $\big(U, \tau_R(X)\big)$. This implies that $M \cap Z^C \neq \emptyset$, which is contradictory to our assumption. Therefore, $b \in$ *Nrwcl(M)*.

Theorem 3.3 Let $f: \big(P, \tau_R(S)\big) \to (Q, \tau_R(T))$ be a function. Then the following are equivalent:

(i) For each *b* in *P*, and each neighbourhood *R*of *f(b)*, there is a Nrw neighbourhood *N* of *b* such that *f(N)* $\subseteq$ *R*.

(ii) For each subset *M*of $\big(P, \tau_R(S)\big)$, *f(Nrwcl (M))*$\subseteq$*Ncl f (M))*.

Proof Suppose (i) holds and let $k \in$*f(Nrwcl(M))* and let R be the neighbourhood of k. Then there exists, $b \in P$ and a Nrw-neighbourhood N of b such that *f(b)=k*, $b \in N, b \in Nrwcl(M) and f(N) \subseteq R$. Since $b \in Nrwcl(M), by theorem$ 4.2 $N \cap M \neq \emptyset$ and hence $[f(M) \cap R] \neq \emptyset$. Hence $k = f(b) \in Ncl[f(M)]. i.e., f(Nrwcl(M) \subseteq Ncl[f(M\ \ )]$. On the otherside, let us assume that (ii) holds and let $b \in P$ and R to be the neighbourhood of *f(b)*. Let $M = f^{-1}({}^{Q}/_{R})$. By the hypothesis, $f(Nrwcl(M) \subseteq Ncl[f(M\ \ )] \subseteq (Q/R\ \ )$ then *Nrwcl(M)=M*. Since b does not belong to *Nrwcl(M)*, there exists a Nrw-neighbourhood N of b such that $N \cap M = \emptyset$ and hence $f(N) \subseteq f({}^{P}/_{M}) \subseteq R$.

Theorem 3.4 Let $f: \left(P, \tau_R(S)\right) \to (Q, \tau_R(T))$ be a function. Then the following are equivalent:
(i) The inverse of each Nano open set in $(Q, \tau_R(T))$ is Nrw-open in $\left(P, \tau_R(S)\right)$.
(ii) For each b in P, the inverse of every neighbourhood of *f(b)* is a Nano neighbourhood of b.
Proof For $b \in \left(P, \tau_R(S)\right)$, let R be the neighbourhood of *f(b)*. Then there exists a Nano-open set Y in $\left(Q, \tau_R(T)\right)$such that *f(b)*$\in Y \subseteq R$. Consequently, $f^{-1}(Y)$ is Nrw-open set in $\left(P, \tau_R(S)\right)$and $b \in f^{-1}(Y) \subseteq f^{-1}(R)$. Here $f^{-1}(R)$ is Nrw-neighbourhood in $\left(P, \tau_R(S)\right)$.

Theorem 3.5 Let $f: \left(P, \tau_R(S)\right) \to (Q, \tau_R(T))$ be a function then the following are equivalent:
(iii) For each b in P, the inverse of every neighbourhood of *f(b)* is a Nano neighbourhood of b.
(iv) For each b in P, and each neighbourhood $R$of *f(b)*, there is a Nrw-neighbourhood N of b such that *f(N)* $\subseteq R$.
Proof Suppose that $b \in P and R be the neighbourhood$ of *f(b)*. By hypothesis, $N = f^{-1}(R)$ is a Nrw-neighbourhood of b. Then $f(N) = f[f^{-1}(R)] \subseteq R$.

Theorem 3.6 For each b in $\left(P, \tau_R(S)\right)$ and for each neighbourhood $R$of *f(b)*, if there is a Nrw-neighbourhood N of b such that *f(N)* $\subseteq R$, then for each point b in $\left(P, \tau_R(S)\right)$ and each Nano-open set R in with $(Q, \tau_R(T))$ with *f(b)* $\in R$, there is a Nrw-open set Y in P such that $b \in Y, f(Y) \subseteq R$.

Proof For $b$in $\left(P, \tau_R(S)\right)$, let R be a Nano-open set containing *f(b)*. Then R is a neighbourhood of *f(b)*. By the hypothesis, there exists Nrw-neighbourhood N of b, such that *f(N)* $\subseteq R$. Thus there exists a Nrw-open set Y in P such that $b \in Y \subseteq N$. Then $f(Y) \subseteq f(N) \subseteq R$. Hence $f(Y) \subseteq R$.

4. NANO REGULAR WEAKLY CLOSED MAPS

Definition 4.1 A map $f: \left(P, \tau_R(S)\right) \to (Q, \tau_R(T))$ is said to be Nrw-closed map, if the image of every Nano closed set in P is Nrw-closed set in Q

Example 4.2 Let $P = \{a, b, c, d\}$ with $P/R = \{\{a\}, \{b\}, \{c\}, \{b, d\}\}$ and $S = \{a, b\}$ Then $\tau_R(S) = \{\emptyset, P, \{a\}, \{b, d\}, \{a, b, d\}\}$ Let $Q = \{k, l, m, n\}$ with $Q/R = \left\{\{k\}, \{l\}, \{m, n\}\right\}$ and $T = \{k, m\}$. Then $\tau_R(T) = \left\{\emptyset, Q, \{k\}, \{m, n\}, \{k, m, n\}\right\}$ .Define $f: \left(P, \tau_R(S)\right) \to \left(Q, \tau_R(T)\right)$ as $f(a) = m, f(b) = k, f(c) = l, f(d) = n.$

Definition 4.3 Let $f: \left(P, \tau_R(S)\right) \to (Q, \tau_R(T))$ be a function. Then f is said to be strongly Nrw-closed map, if the image of every Nrw-closed set in P is Nrw-closed in Q.

Definition 4.4 A function $f: \left(P, \tau_R(S)\right) \to (Q, \tau_R(T))$ is said to be Nrw-almost closed map, if the image of every Nrw-closed set in P is Nrw-closed in Q.

Theorem 4.5 A mapping $f: \left(P, \tau_R(S)\right) \to (Q, \tau_R(T))$ is Nrw-closed map if and only if $Nrwcl(f(M)) \subseteq f(Ncl(M))$ for every subset M of P.
Proof Let f be Nrw-closed and $M \subseteq P$. Since *Ncl(M)* is Nrw-closed in P. Then *f(Ncl(M))* is Nrw-closed in $Q. Then f(M) \subseteq f\left(Ncl(M)\right)$ because $A \subseteq Ncl(A)$. From Corollary (3.1.4) and (3.1.5), we have $Nrwcl(f(M)) \subseteq Nrwcl(f(Ncl(M))) \subseteq f\left(Ncl(M)\right)$. Hence $Nrwcl(f(M)) \subseteq f(Ncl(M))$. (check for corollary)

Remark 4.6 The subsequent implication explains the relationship between different types of Nano-closed maps and Nano-open maps. The following example shows that, the reverse of this need not be true generally.

Example 4.7 Let $P = \{a, b, c, d, e\}$with $P/R = \{\{d\}, \{a, b\}, \{c, e\}\}$ and $S = \{a, d\}$ Then $\tau_R(S) = \{\emptyset, P, \{d\}, \{a, b\}, \{a, b, d\}\}$ Let $Q = \{k, l, m, n, o\}$ with $Q/R = \{\{k\}, \{l, m\}, \{n, o\}\}$ and $T = \{k, l\}$. Then $\tau_R(T) = \{\emptyset, Q, \{k\}, \{l, m\}, \{k, l, m\}\}$. Define $f: (P, \tau_R(S)) \to (Q, \tau_R(T))$ as $f(a) = l, f(b) = m, f(c) = f(d) = n, f(e) = o.$
Hence the function f is Nano closed map and also Nrw-closed map. But it is not Nrw-almost closed map as well as not strongly Nrw-closed map.

Example 4.8 Let $P = \{a, b, c, d, e\}$ with $P/R = \{\{a\}, \{b, c\}, \{d, e\}\}$ and $S = \{a, b\}$. Hence $\tau_R(S) = \{\emptyset, P, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let Q = $\{k, l, m, n, o\}$ with $Q/R = \{\{n\}, \{k, l\}, \{m, o\}\}$ and $T = \{k, n\}\tau_R(T) = \{\emptyset, Q, \{n\}, \{k, l\}, \{k, l, n\}\}$ Define $f: (P, \tau_R(S)) \to (Q, \tau_R(T))$ as $f(a) = m, f(b) = f(c) = o, f(d) = n, f(e) = l.$
Then the function f is Nrw-closed map and Nrw-almost closed map. But it is neither strongly Nrw-closed map nor Nrw-closed map.

Example 4.9 Let $P = \{a, b, c, d, e\}$with $P/R = \{\{d\}, \{a, b\}, \{c, e\}\}$ and $S = \{a, d\} then \tau_R(S) = \{\emptyset, P, \{d\}, \{a, b\}, \{a, b, d\}\}$Let $Q = \{k, l, m, n, o\}$ with $Q/R = \{\{k\}, \{l, m\}, \{n, o\}\}$and $T = \{k, l\}$. Then $\tau_R(T) = \{\emptyset, Q, \{k\}, \{l, m\}, \{k, l, m\}\}$. Define $f: (P, \tau_R(S)) \to (Q, \tau_R(T))$ as $f(a) = l, f(b) = m, f(c) = o, f(d) = f(e) = k.$
Then function f isNrw-open map and also nano open map. But it is not Nrw-almost open map as well as not strongly Nrw-open map.

Example 4.10 Let $P = \{a, b, c, d, e\}$ with $P/R = \{\{a\}, \{b, c\}, \{d, e\}\}$and $S = \{a, b\}$. hence $\tau_R(S) = \{\emptyset, P, \{a\}, \{b, c\}, \{a, b, c\}\}$. Let Q = $\{k, l, m, n, o\}$ with $Q/R = \{\{n\}, \{k, l\}, \{m, o\}\}$ and $T = \{k, n\}$, $\tau_R(T) = \{\emptyset, Q, \{n\}, \{k, l\}, \{k, l, n\}\}$ Define $f: (P, \tau_R(S)) \to (Q, \tau_R(T))$ as $f(a) = l, f(b) = f(e) = n, f(c) = f(d) = k.$
This function f is Nrw-open map and Nrw-almost open map. But it is neither strongly Nrw-open map nor Nrw-open map.

5. CONCLUSION

In this paper we have discussed the concepts of Nano regular weakly neighbourhood and Nano regular weakly closed map. These concepts can be used to derive a new real world applications in future.

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