Solving QAP with large size 10 facilities and 10 locations

[1]Haleemah jawad kadhim, [2]Mushtak A. K. Shiker

[1]*Department of mathematics, collage of education for pure sciences, university of Babylon, Babil- Iraq,* halima.kazem@student.uobabylon.edu.iq
[2]*Department of mathematics, collage of education for pure sciences, university of Babylon, Babil- Iraq.*

Abstract

The object of the QAP is to allocate a set of locations to a set of facilities, where the cost is the function of the flow and the distance among the facilities. The main objective of QAP is to minimize the cost by assigning each facility to a location where the costs is the sum of all possible flow- distance products. In this work we applied QAP to solve several problems with diverse number of flows, especially the problem with 10 facilities and 10 locations.

Keywords: Quadratic assignment. Facilities planning. Permutation matrices. Position theory.

INTRODUCTION

In position theory, QAP (quadratic assignment problem) is considered as a well-known problem, it was suggested by Koopmans and Beckmaun in the fifties of the last century as a mathematical model to locate set of indivisible economic activities [1]. Since that, many papers have been introduced to solve QAP. In 1961 [2], Steinberg employed it to minimize the number of connections between components in a backboard wiring. In 1968 [3], Nugent et al. introduced an experimental comparison of techniques for the assignment of facilities to locations. In 2003 [4], Anstreicher introduced a new technique to solve it. In recent years many authors working on a new techniques for assignment problem, for example, in 2020, Hussein and Shiker introduced several papers to solve it [5- 7].

Lawler [8] introduced the following general type of QAP :

$$\text{QAP} = \min \sum_{i,j,k,l} d_{ijkl} x_{ij} x_{kl} \qquad (1)$$

$$\text{s.t. } x \in \varphi ,$$

were φ represents the set of n×n permutation matrices. As it show from (1), the cost of QAP is the sum of the flow between a pair of facilities multiplied by the distance between their assigned locations over all pairs. In practice a lot of QAPs have restricted "Koopmans-Beckmann" shape that is more constrained, corresponding with d_ijkl =a_ijkl, i=k, j=l. In the application of a facility venue x_ij=1 corresponds to facility i being positioned in position j. The flow between installation i and k is a_ik, and the distance between sites j and l is b_jl.

2. Koopmans and Beckmann formulation [3]

Koopmans and Beckmann introduced the QAP formula.

Definition Let X=(x_ij) be a matrix with n x n size. If x_ij satisfies:

$$\sum_{i=1}^{n} x_{ij} = 1, \ \ 1 \le j \le n \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij} = 1, \ \ 1 \le i \le n$$

$$x_{ij} \in \{0,1\}, \quad 1 \le i,j \le n$$

then X is a permutation matrix. The set of all n x n permutation matrices is denoted by Π_n.

QAP (A,B) is equivalent to :

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} a_{ij}b_{kl}x_{ik}x_{jl} \qquad (3)$$

S.t.

$$\sum_{i=1}^{n} x_{ij} = 1, \ \ 1 \le j \le n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \ \ 1 \le i \le n$$

$$x_{ij} \in \{0,1\}, \quad 1 \le i,j \le n$$

(3) is called Koopmans-Beckmann formulation of QAP. If we want to minimize a function over x_ij≤i,j≤n, satisfying (2), we get:

$$min \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad (4)$$

S. t.

$$\sum_{i=1}^{n} x_{ij} = 1, \ \ 1 \le j \le n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \ \ 1 \le i \le n$$

$$x_{ij} \in \{0,1\}, \quad 1 \le i,j \le n$$

where 〖C=c〗_ij is an n x n cost matrix.

The framework of solving QAPs

Consider 4 facilities (f) and 4 locations (l) as a problem of facility location. If f2 is assigned to l1, f1 is assigned to l2, f4 is assigned to l3, and f3 is assigned to l4. We can write that as p={2,1,4,3}, where p is the permutation, that means f2 is assigned to l1, f1 is assigned to l2, f4 is assigned to l3, and f3 is assigned to l4. Any line between any two facilities shows there is a flow between these two facilities, so that, when the flow is more increasing between any two facilities, the line between them will be more thickness. The authors worked on finding the solutions in variant fields such as optimization, reliability, transportation problems, and so on. For examples, to find the optimal solution of nonlinear systems and optimization problems we used the projection technique [9- 15], trust region techniques [16- 20], conjugate gradient techniques [21- 24], and line search techniques [25- 28], and some article in reliability [29- 34], but in this work we employed QAP to solve the following examples and get their optimal solutions, these examples beginning from size 4 to the large size 10.

QAP of size 4

Assign each facility (1,2,3,4) to one location (A,B,C,D).

A=1 ,B=2 ,C=3 ,D=4

Optimal Solution =471

Table 1 : *Flows between facilities of size 4*

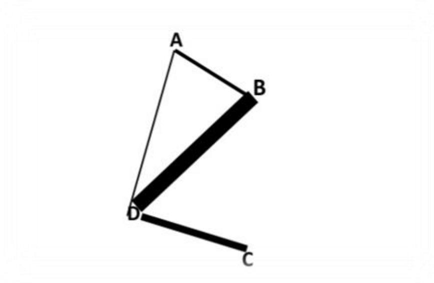
Flows				
	1	**2**	**3**	**4**
1	0	2	0	1
2	2	0	0	4
3	0	0	0	3
4	1	4	3	0

Table 2 : *Distance between facilities of size 4*

Distance Matrix				
	A	**B**	**C**	**D**
A	0	15	51	51
B	15	0	20	60
C	51	20	0	50
D	51	60	50	0

$0X_1 + 15X_2 + 51X_3 + 51X_4$
$15X_1 + 0X_2 + 20X_3 + 60X_4$
$51X_1 + 20X_2 + 0X_3 + 50X_4$
$51X_1 + 60X_2 + 50X_3 + 0X_4$

$0Y_1 + 2Y_2 + 0Y_3 + 1Y_4$
$2Y_1 + 0Y_2 + 0Y_3 + 4Y_4$
$0Y_1 + 0Y_2 + 0Y_3 + 3Y_4$
$1Y_1 + 4Y_2 + 3Y_3 + 0Y_4$



Fig(1): *The QAP of size 4*

QAP of size 5

Assign each facility (1,2,3,4,5) to one location (A,B,C,D,E).

A=1,B = 2 ,C=3 ,D=4 ,E=5

Optimal solution =312

Table 3 : *Distance between facilities of size 5*

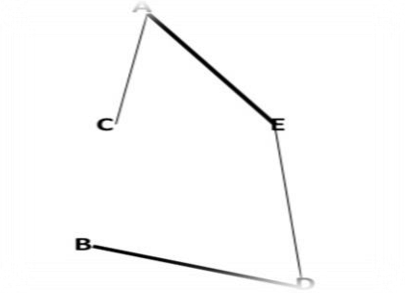
Distance Matrix					
	A	**B**	**C**	**D**	**E**
A	0	52	52	90	52
B	52	0	20	52	30
C	52	20	0	40	12
D	90	52	40	0	52
E	52	30	12	52	0

Table 4 : *Flows between facilities of size 5*

Flows					
	1	**2**	**3**	**4**	**5**
1	0	0	1	0	2
2	0	0	0	2	0
3	1	0	0	0	0
4	0	2	0	0	1
5	2	0	0	1	0

$0X_1 + 52X_2 + 52X_3 + 90X_4 + 52X_5$
$52X_1 + 0X_2 + 20X_3 + 52X_4 + 30X_5$
$52X_1 + 20X_2 + 0X_3 + 40X_4 + 12X_5$
$90X_1 + 52X_2 + 40X_3 + 0X_4 + 52X_5$
$52X_1 + 30X_2 + 12X_3 + 52X_4 + 0X_5$

$0Y_1 + 0Y_2 + 1Y_3 + 0Y_4 + 2Y_5$
$0Y_1 + 0Y_2 + 0Y_3 + 2Y_4 + 0Y_5$
$1Y_1 + 0Y_2 + 0Y_3 + 0Y_4 + 0Y_5$
$0Y_1 + 2Y_2 + 0Y_3 + 0Y_4 + 1Y_5$
$2Y_1 + 0Y_2 + 0Y_3 + 1Y_4 + 0Y_5$



Fig(2): *The QAP of size 5*

QAP of size 6

Assign each facility (1,2,3,4,5,6) to one location (A,B,C,D,E,F).

A=1 ,B=2 ,C=3 ,D=4 ,E=5,F=6

Optimal solution =470

Table 5 : *Distance between facilities of size 6*

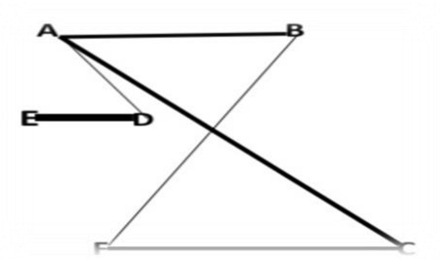
Distance Matrix						
	A	**B**	**C**	**D**	**E**	**F**
A	0	42	60	30	20	62
B	42	0	40	20	30	70
C	60	40	0	20	40	50
D	30	20	20	0	22	52
E	20	30	40	22	0	40
F	62	70	50	52	40	0

Table 6 : *Flows between facilities of size 6*

Flows						
	1	**2**	**3**	**4**	**5**	**6**
1	0	2	2	1	0	0
2	2	0	0	0	0	1
3	2	0	0	0	0	2
4	1	0	0	0	3	0
5	0	0	0	3	0	0
6	0	1	2	0	0	0

$$0X_1 + 42X_2 + 60X_3 + 30X_4 + 20X_5 + 62X_6$$
$$42X_1 + 0X_2 + 40X_3 + 20X_4 + 30X_5 + 70X_6$$
$$60X_1 + 40X_2 + 0X_3 + 20X_4 + 40X_5 + 50X_6$$
$$30X_1 + 20X_2 + 20X_3 + 0X_4 + 22X_5 + 52X_6$$
$$20X_1 + 30X_2 + 40X_3 + 22X_4 + 0X_5 + 40X_6$$
$$62X_1 + 70X_2 + 50X_3 + 52X_4 + 40X_5 + 0X_6$$

$$0Y_1 + 2Y_2 + 2Y_3 + 1Y_4 + 0Y_5 + 0Y_6$$
$$2Y_1 + 0Y_2 + 0Y_3 + 0Y_4 + 0Y_5 + 1Y_6$$
$$2Y_1 + 0Y_2 + 0Y_3 + 0Y_4 + 0Y_5 + 2Y_6$$
$$1Y_1 + 0Y_2 + 0Y_3 + 0Y_4 + 3Y_5 + 0Y_6$$
$$0Y_1 + 0Y_2 + 0Y_3 + 3Y_4 + 0Y_5 + 0Y_6$$
$$0Y_1 + 1Y_2 + 2Y_3 + 0Y_4 + 0Y_5 + 0Y_6$$



Fig(3): *The QAP of size 6*

QAP of size 7

Assign each facility (1,2,3,4,5,6,7) to one location (A,B,C,D,E,F,G).

A=1 ,B=2 ,C=3 ,D=4 ,E=5,F=6 ,G=7

Optimal Solution =612

Table 7 : *Distance between facilities of size 7*

Distance Matrix							
	A	**B**	**C**	**D**	**E**	**F**	**G**
A	0	35	71	99	71	75	41
B	35	0	42	80	65	82	47
C	71	42	0	45	49	79	55
D	99	80	45	0	36	65	65
E	71	65	49	36	0	31	32
F	75	82	79	65	31	0	36
G	41	47	55	65	32	36	0

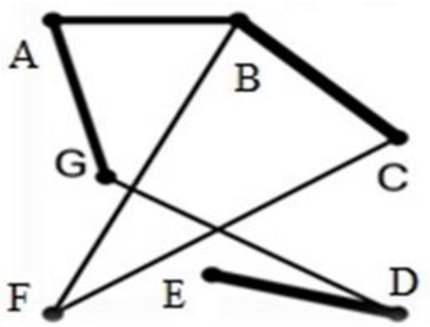
Table 8 : *Flows between facilities of size 7*

Flows Matrix							
	1	**2**	**3**	**4**	**5**	**6**	**7**
1	0	2	0	0	0	0	2
2	2	0	3	0	0	1	0
3	0	3	0	0	0	1	0
4	0	0	0	0	3	0	1
5	0	0	0	3	0	0	0
6	0	1	1	0	0	0	0
7	2	0	0	1	0	0	0

$$0X_1 + 35X_2 + 71X_3 + 99X_4 + 71X_5 + 75X_6 + 41X_7$$
$$35X_1 + 0X_2 + 42X_3 + 80X_4 + 65X_5 + 82X_6 + 47X_7$$

$$71X_1 + 42X_2 + 0X_3 + 45X_4 + 49X_5 + 19X_6 + 55X_7$$
$$99X_1 + 80X_2 + 45X_3 + 0X_4 + 36X_5 + 65X_6 + 65X_7$$
$$71X_1 + 65X_2 + 49X_3 + 36X_4 + 0X_5 + 31X_6 + 32X_7$$
$$75X_1 + 82X_2 + 19X_3 + 65X_4 + 31X_5 + 0\,X_6 + 36X_7$$
$$41X_1 + 47X_2 + 55X_3 + 65X_4 + 32X_5 + 36X_6 + 0X_7$$

$$0Y_1 + 2Y_2 + 0Y_3 + 0Y_4 + 0Y_5 + 0Y_6 + 2Y_7$$
$$2Y_1 + 0Y_2 + 3Y_3 + 0Y_4 + 0Y_5 + 1Y_6 + 0Y_7$$
$$0Y_1 + 3Y_2 + 0Y_3 + 0Y_4 + 0Y_5 + 1Y_6 + 0Y_7$$
$$0Y_1 + 0Y_2 + 0Y_3 + 0Y_4 + 3Y_5 + 0Y_6 + 1Y_7$$
$$0Y_1 + 0Y_2 + 0Y_3 + 3Y_4 + 0Y_5 + 0Y_6 + 0Y_7$$
$$0Y_1 + 1Y_2 + 0Y_3 + 0Y_4 + 0Y_5 + 0Y_6 + 0Y_7$$
$$2Y_1 + 0Y_2 + 0Y_3 + 1Y_4 + 0Y_5 + 0Y_6 + 0Y_7$$



Fig(4): *The QAP of size 7*

The following example is more complex QAP with larger size =10.

QAP of size 10

Assign each facility (1,2,3,4,5,6,7,8,9,10) to one location (A,B,C,D,E,F,G,H,I,J).

A=1,B=2 ,C=3 ,D=4 ,E=5,F=6 ,G=7 ,H=8 ,I=9 ,J=10

Optimal Solution =2299

Table 9 : *Distance between facilities of size 10*

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
A	0	30	62	90	72	70	71	70	31	20
B	30	0	41	70	52	42	53	80	42	42
C	62	41	0	41	12	45	70	52	50	50
D	90	70	41	0	30	33	32	65	70	19
E	72	52	12	30	0	31	30	15	60	53
F	70	42	45	33	31	0	15	32	36	40
G	71	53	70	32	30	15	0	15	53	19
H	70	80	52	65	15	32	15	0	55	24
I	31	42	50	70	60	36	53	55	0	22
J	20	42	50	19	53	40	19	24	22	0

Table 10 : *Flows between facilities of size 10*

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	0	1	2	4	0	0	0	2	0	0
2	1	0	6	1	0	3	0	0	0	2
3	2	6	0	0	0	0	2	0	0	2
4	4	1	0	0	0	0	1	2	1	0
5	0	0	0	0	0	2	0	1	2	0
6	0	3	0	0	2	0	0	0	0	0
7	0	0	2	1	0	0	0	0	2	4
8	2	0	0	2	1	0	0	0	4	2
9	0	0	0	1	2	0	2	4	0	0
10	0	2	2	0	0	0	4	2	0	0

$$0X_1 + 30X_2 + 62X_3 + 90X_4 + 72X_5 + 70X_6 + 71X_7 + 70X_8 + 31X_9 + 20X_{10}$$

$$30X_1 + 0X_2 + 41X_3 + 70X_4 + 52X_5 + 42X_6 + 53X_7 + 80X_8 + 42X_9 + 42X_{10}$$

$$62X_1 + 41X_2 + 0X_3 + 41X_4 + 12X_5 + 45X_6 + 70X_7 + 52X_8 + 50X_9 + 50X_{10}$$

$$90X_1 + 70X_2 + 41X_3 + 0X_4 + 30X_5 + 33X_6 + 32X_7 + 65X_8 + 70X_9 + 19X_{10}$$

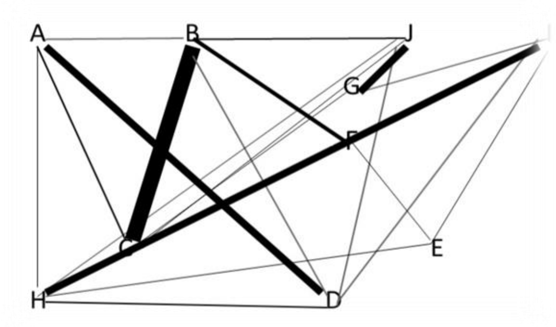
$$72X_1 + 52X_2 + 12X_3 + 30X_4 + 0X_5 + 31X_6 + 30X_7 + 15X_8 + 60X_9 + 53X_{10}$$

$$70X_1 + 42X_2 + 45X_3 + 33X_4 + 31X_5 + 0X_6 + 15X_7 + 32X_8 + 36X_9 + 40X_{10}$$

$$71X_1 + 53X_2 + 70X_3 + 32X_4 + 30X_5 + 15X_6 + 0X_7 + 15X_8 + 53X_9 + 19X_{10}$$

$$70X_1 + 80X_2 + 52X_3 + 65X_4 + 15X_5 + 32X_6 + 15X_7 + 0X_8 + 55X_9 + 24X_{10}$$
$$31X_1 + 42X_2 + 50X_3 + 70X_4 + 60X_5 + 36X_6 + 53X_7 + 55X_8 + 0X_9 + 22X_{10}$$
$$20X_1 + 42X_2 + 50X_3 + 19X_4 + 53X_5 + 40X_6 + 19X_7 + 24X_8 + 22X_9 + 0X_{10}$$

$$0Y_1 + 1Y_2 + 2Y_3 + 4Y_4 + 0Y_5 + 0Y_6 + 0Y_7 + 2Y_8 + 0Y_9 + 0Y_{10}$$
$$1Y_1 + 0Y_2 + 6Y_3 + 1Y_4 + 0Y_5 + 3Y_6 + 0Y_7 + 0Y_8 + 0Y_9 + 2Y_{10}$$
$$2Y_1 + 6Y_2 + 0Y_3 + 0Y_4 + 0Y_5 + 0Y_6 + 2Y_7 + 0Y_8 + 0Y_9 + 2Y_{10}$$
$$4Y_1 + 1Y_2 + 0Y_3 + 0Y_4 + 0Y_5 + 0Y_6 + 1Y_7 + 2Y_8 + 1Y_9 + 0Y_{10}$$
$$0Y_1 + 0Y_2 + 0Y_3 + 0Y_4 + 0Y_5 + 2Y_6 + 0Y_7 + 1Y_8 + 2Y_9 + 0Y_{10}$$
$$6Y_1 + 0Y_2 + 3Y_3 + 0Y_4 + 0Y_5 + 2Y_6 + 0Y_7 + 0Y_8 + 0Y_9 + 0Y_{10}$$
$$0Y_1 + 0Y_2 + 2Y_3 + 1Y_4 + 0Y_5 + 0Y_6 + 0Y_7 + 0Y_8 + 2Y_9 + 4Y_{10}$$
$$2Y_1 + 0Y_2 + 0Y_3 + 2Y_4 + 1Y_5 + 0Y_6 + 0Y_7 + 0Y_8 + 4Y_9 + 2Y_{10}$$
$$0Y_1 + 0Y_2 + 0Y_3 + 1Y_4 + 2Y_5 + 0Y_6 + 2Y_7 + 4Y_8 + 0Y_9 + 0Y_{10}$$
$$0Y_1 + 2Y_2 + 2Y_3 + 0Y_4 + 0Y_5 + 0Y_6 + 4Y_7 + 2Y_8 + 0Y_9 + 0Y_{10}$$



Fig(5): *The QAP of size 10*

Conclusion

Until now, QAP is considered as most difficult combinatorial optimization problems, however, many researchers are working on it because of its many day life applications. The remarkable and amazing development in programming and computing processes makes it easy to work with QAP in spite of the hardness of dealing with it.

In this work we employed QAP to find the optimal solution for several examples with small size and large size of facilities, and then we drew the solutions figures.

Reference

[1] Koopmans T C and Beckmann M J 1957 Assignment problems and the location of economic activities, Econometrica, 25, pp. 53–76.

[2] Steinberg L 1961The backboard wiring problem, A placement algorithm. SIAM Rev. 3, pp. 37–50,.

[3] Nugent C E, Vollman T E, and Ruml J 1968 An experimental comparison of techniques for the assignment of facilities to locations. Oper. Res. 16, pp. 150–173.

[4] Anstreicher K M 2003 Recent advances in the solution of quadratic assignment problems. Mathematical Programming. 97, pp. 27–42.

[5] Hussein H A and Shiker M A K 2020 A modification to Vogel's approximation method to Solve transportation problems, J. Phys.: Conf. Ser. 1591 012029.

[6] Hussein H A and Shiker M A K 2020 Two New Effective Methods to Find the Optimal Solution for the Assignment Problems, Journal of Advanced Research in Dynamical and Control Systems, 12: 7, p 49- 54.

[7] Hussein H A, Shiker M A K and Zabiba M S M 2020 A new revised efficient of VAM to find the initial solution for the transportation problem, J. Phys.: Conf. Ser. 1591 012032.

[8] Lawler E L 1963 The quadratic assignment problem. Mgmt. Sci. 9, 586–599.

[9] Shiker M A K and Amini K 2018 A new projection-based algorithm for solving a large scale nonlinear system of monotone equations, Croatian operational research review, 9: 1, p 63-73.

[10] Mahdi M M et al. 2021 Solving systems of nonlinear monotone equations by using a new projection approach, J. Phys.: Conf. Ser. 1804 012107

[11] Mahdi M M and Shiker M A K 2020 A new projection technique for developing a Liu-Storey method to solve nonlinear systems of monotone equations, J. Phys.: Conf. Ser. 1591 012030.

[12] Mahdi M M and Shiker M A K 2020 Three terms of derivative free projection technique for solving nonlinear monotone equations, J. Phys.: Conf. Ser. 1591 012031.

[13] Mahdi M M and Shiker M A K 2020 A New Class of Three-Term Double Projection Approach for Solving Nonlinear Monotone Equations, J. Phys.: Conf. Ser. 1664 012147.
[14] Dreeb N K, et al. 2021, Using a New Projection Approach to Find the Optimal Solution for Nonlinear Systems of Monotone Equation, J. Phys.: Conf. Ser. 1818 012101.
[15] Mahdi M M and Shiker M A K 2020 Three-Term of New Conjugate Gradient Projection Approach under Wolfe Condition to Solve Unconstrained Optimization Problems, Journal of Advanced Research in Dynamical and Control Systems, 12: 7, p 788- 795.
[16] Shiker M A K & Sahib Z 2018 A modified trust-region method for solving unconstrained optimization. Journal of Engineering and Applied Sciences, 13: 22, p 9667– 9671.
[17] Dwail H H and Shiker M A K 2020 Reducing the time that TRM requires to solve systems of nonlinear equations, IOP Conf. Ser.: Mater. Sci. Eng. 928 042043
[18] Dwail H H and Shiker M A K 2020 Using a trust region method with nonmonotone technique to solve unrestricted optimization problem, J. Phys.: Conf. Ser. 1664 012128.
[19] Dwail H H et al. 2021 A new modified TR algorithm with adaptive radius to solve a nonlinear systems of equations, J. Phys.: Conf. Ser. 1804 012108.
[20] Dwail H H and Shiker M A K 2021 Using trust region method with BFGS technique for solving nonlinear systems of equations, J. Phys.: Conf. Ser. 1818 012022.
[21] Wasi H A and Shiker M A K 2021 Proposed CG method to solve unconstrained optimization problems, J. Phys.: Conf. Ser. 1804 012024.
[22] Wasi H A and Shiker M A K 2020 A new hybrid CGM for unconstrained optimization problems, J. Phys.: Conf. Ser. 1664 012077.
[23] Wasi H A and Shiker M A K 2021 Nonlinear conjugate gradient method with modified Armijo condition to solve unconstrained optimization, J. Phys.: Conf. Ser. 1818 012021.
[24] Wasi H A and Shiker M A K 2021 A modified of FR method to solve unconstrained optimization, J. Phys.: Conf. Ser. 1804 012023.
[25] Hashim K H and Shiker M A K 2021 Using a new line search method with gradient direction to solve nonlinear systems of equations, J. Phys.: Conf. Ser. 1804 012106.
[26] Hashim K H, et al. 2021 Solving the Nonlinear Monotone Equations by Using a New Line Search Technique, J. Phys.: Conf. Ser. 1818 012099.
[27] Hashim L H, et al. 2021 An application comparison of two negative binomial models on rainfall count data, J. Phys.: Conf. Ser. 1818 012100.
[28] Hashim L H, et al. 2021 An application comparison of two Poisson models on zero count data, J. Phys.: Conf. Ser. 1818 012165.
[29] Hassan Z A H and Mutar E K 2017 Geometry of reliability models of electrical system used inside spacecraft, Second Al-Sadiq International Conference on Multidisciplinary in IT and Communication Science and Applications (AIC-MITCSA), pp. 301-306.
[30] Hassan Z A H and Balan V 2017 Fuzzy T-map estimates of complex circuit reliability, International Conference on Current Research in Computer Science and Information Technology (ICCIT-2017), IEEE, Special issue, pp. 136-139.
[31] Hassan Z A H and Balan V 2015 Reliability extrema of a complex circuit on bi-variate slice classes, Karbala International Journal of Modern Science, 1: 1, pp. 1- 8.
[32] Hassan Z A H and Shiker M A K 2018 Using of generalized baye's theorem to evaluate the reliability of aircraft systems. Journal of Engineering and Applied Sciences, (Special Issue13), 10797–10801.
[33] Abdullah G and Hassan Z A H 2020 Using of particle swarm optimization (PSO) to addressed reliability allocation of complex network, J. Phys.: Conf. Ser. 1664: 012125.
[34] Abdullah G and Hassan Z A H 2020 Using of Genetic Algorithm to Evaluate Reliability Allocation and Optimization of Complex Network, IOP Conf. Ser.: Mater. Sci. Eng. 928: …