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Stochastic Programming for Agricultural Planning

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Abstract:

Problems related to agriculture are, in essence, stochastic because of the uncertain nature of their parameters. The uncertainty caused by factors such as climatic conditions on yield impacts many systems arising in this sector. Uncertainty and imperfect information involved therein are challenging decision-making, as decision-makers are led to make decisions before observing the realization of the random factors.

Traditional approaches to deal with agricultural problems do not integrate risks and uncertainties involved therein, while it is relevant to efficient managerial decision-making to consider uncertainties and respond to opportunities and threats. Stochastic optimization has been a key to solving problems related to agriculture and enhancing productivity and efficiency in this field. It helps manage uncertainty and provides robust solutions. Stochastic optimization provides decision-makers with the ability to make optimal management decisions and helps to minimize the costs associated with decision-making under uncertainty.

This paper focuses on stochastic programming and covers some of the theoretical foundations. It also focuses on recent advances in agriculture as an area where stochastic programming is applicable.

Keywords: Stochastic optimization- Stochastic approaches - Agriculture- Uncertainty.

1 INTRODUCTION

The agricultural sector plays a strategic role in the economic development of each country. It is of irrevocable economic importance and key for economic transformation and food security. The performance of the sector, combined with its interactions with other economic sectors, gives it a prominent place.

The agricultural sector makes its contribution to economic development in several ways. It generates economic opportunities and delivers significant support to employment policy, especially in rural areas. It is also key to reducing poverty, particularly in developing countries.

This field is subject to significant mathematical developments to consolidate its strategic vocation and ensure better management of financial resources, risks, and climate shocks. Several mathematical models helped to formalize inherent uncertainty to this sector, and over the years, various approaches to agricultural optimization under uncertainty were developed. This article attempts to present a brief review of the usage of stochastic programming in the agricultural sector.

2 STOCHASTIC OPTIMIZATION

Many concrete problems from different fields can be modeled using mathematical programs. Mathematical programming refers to mathematical models used to solve problems and make optimal decisions. It can seek to maximize or minimize an objective function, often subject to a set of constraints.

More precisely, the formulation of an optimization problem involves:

1. Selecting optimization variables

2. Defining the objective function

3. Identifying the set of constraints

The general model can be represented as:

$$\min\{ f(x) \,/ g(x,a) \geq 0\} \quad (1.1)$$

Where f is the objective function, x is a decision vector, a is a vector of model parameters, and g is the constraint mapping.

Model parameters can be unknown or uncertain and may imply randomness. Historically, uncertainty has been considered in mathematical models for decision-making. Many approaches have been proposed to address this issue. One of the widely used approaches is stochastic programs, as they have proved their flexibility and usefulness in different areas of science. In fact, stochastic models rely on solid mathematical foundations, probability theory, and stochastic processes.

Stochastic optimization is the mathematical framework to model decision-making under uncertainty. It refers to a collection of methods for minimizing or maximizing an objective function when randomness is present. Randomness usually enters the problem through the objective function and/or the constraint set.

Stochastic optimization was first introduced by George B. Dantzig by considering cases that included uncertainty. Over the past few decades, this field has been an active area of research. With the increase of theoretical and algorithmic developments, stochastic programming has been applied to a wide variety of problems, including water resources planning and management, production planning, and financial problems. It plays an important role in the analysis, design, and performance of modern systems.

An important number of optimization problems involving uncertainty occur if the constraints g depend on a stochastic parameter ξ, such as the system in (1.1) might be:

$$\min\{f(x) \mid g(x,\xi) \geq 0\} \qquad (1.2)$$

Here, ξ is a k-dimensional random variable defined on some probability space (Ω, F, P).

To deal with uncertainty and arrive at a relevant and implementable form of the constraints, the dependence of g on specific outcomes of ξ has to be removed. The most prominent approaches to do so are:

- the expected value approach
- the approach by probabilistic constraints:
  - Joint chance constraints
  - Single chance constraints

2.1 Expected value approach

The expected value approach is a common solution procedure to solve stochastic problems. It refers to an optimization method that uses the expected value of the random variables. Thus, to solve the stochastic problem, the easiest and simplest approach is to replace the random variable ξ with its expected value $E(\xi)$, and solve the obtained deterministic problem. The system (1.2) is replaced by:

$$\min\{f(x) \mid g(x, E(\xi)) \geq 0\} \qquad (1.3)$$

The problem (1.3) is an inequality system depending only on the design x. Here, the expectation operator acts as an integrator over ξ.

The main issue with this approach is that sometimes the solution can be inexact or even non-implementable.

It is also possible to use the expected value of the constraints $E(g(x,\xi))$. The stochastic optimization problem becomes:

$$\min\{f(x) \mid E(g(x,\xi)) \geq 0\} \qquad (1.4)$$

The systems (1.3) and (1.4) coincide in case that g depends linearly on ξ.

2.2 Optimization with probabilistic constraints

Often, a design decision x has to be implemented before the parameter ξ is observed. In doing so, it is difficult to find a decision that would definitively exclude subsequent constraints violations caused by unexpected random effects or unexpected extreme events. The choice of x does not guarantee that $g(x,\xi) \geq 0$ for all possible realizations of the parameter ξ.

Stochastic optimization problems with probabilistic constraints, also called stochastic optimization problems with chance constraints, help to deal with uncertainties and obtain a feasible optimal solution. It is often used in case uncertainties are assumed to follow a certain probability distribution.

Probabilistic constraints require that the probability to remain feasible is above a reliability level $p\epsilon[0,1]$. The reliability level is fixed by the decision-maker in order to model the safety requirements.

This approach is one of the most prominent approaches used for dealing with optimization problems under uncertainty.

The problem can be represented as:

$$\min\{f(x)\,|P(g(x,\xi)\geq 0)\geq p\} \qquad p\in[0,1] \quad (1.5)$$

Where:

- x is a decision vector;
- P is the probability measure
- ξ is the random vector
- $p\epsilon[0,1]$ is the prescribed probability or the reliability level. The decision maker prescribes it as a problem parameter.

Higher values of the prescribed probability p lead to a smaller set of feasible decisions.

When dealing with a random inequality system ($g_i(x,\xi)\geq 0 \quad i=(1,\dots,n)$), two options for generating chance constraints are possible:

- Joint chance constraints
- Single chance constraints.

The solution of the chance-constrained problem (1.5) is guaranteed to be a feasible solution to the original problem (1.2).

The chance constraints approach is not too expensive and provides robust solutions. This approach is, however, often difficult to solve.

For a standard reference on probabilistic constraints, it is advisable to refer to the monograph by Prékopa (1995).

2.2.1 Joint chance constraint

To take into consideration the uncertainty, the problem can be formulated as:

$$\min \ f(x) \qquad (1.6)$$

$$\text{s.t.} \quad P(g_i(x,\xi)\geq 0,(\,i=1,\dots,n))\geq p$$

The constraint in (1.6) is a joint chance constraint as it requires reliability in the output feasible region as a whole. The constraints must be maintained at the prescribed level of probability p. It is appropriate to use joint chance constraint in a context where it is important to satisfy all constraints simultaneously. In doing so, a decision x would be considered feasible if and only if the random inequality $g_i(x,\xi)\geq 0$ is satisfied at least with a reliability level $p\epsilon[0,1]$. The prescribed probability p is typically chosen close to one. This approach leads to optimal solutions but involves higher costs.

2.2.2 Individual chance constraints

Individual probabilistic constraints approach calls for reliability in the individual output feasible region. Here, each component of the random inequality system is individually turned into a probabilistic constraint.

An optimization problem with single chance constraints has the typical form:

$$\min \ f(x) \qquad (1.7)$$

$$\text{s.t.} \quad P(g_i(x,\xi)\geq 0)\geq p_i \quad (\,i=1,\dots,n)$$

It is much easier to deal with individual probabilistic constraints, especially in the case where the component $g_i(x,\xi)$ is separable with respect to ξ. In fact, it would be easy to convert these constraints into explicit ones via quantiles.

Furthermore, individual probabilistic constraints guarantee that the probability of constraints violation is low at each fixed time. However, over the whole time interval violation may be likely.

Moreover, it is easy to demonstrate that:

- If x is feasible for (1.6) then x is feasible for (1.7) too provided that $p_i=p \quad \forall i$
- If x is feasible for (1.7) then x is feasible for (1.6) too provided that:

$$\sum_{i=1}^{n} p_i \geq p+n-1$$

Or

$$p_i=\tilde{p} \quad \forall i \qquad \text{and} \qquad \tilde{p}\geq \frac{p+n-1}{n}$$

3 APPLICATIONS OF STOCHASTIC OPTIMIZATION IN AGRICULTURE

Agriculture is subject to uncertainty and risk. It is driven by uncontrollable factors like climate conditions, pests, diseases, weeds, and variability of prices. These factors affect farmland management and imply yield uncertainty. Thus, it is relevant to managerial decision-making to consider uncertainties and respond to opportunities and threats. The decision-maker faces uncertainty and imperfect information when making almost all management decisions in this dynamic environment including investment, plan planting, irrigation, and harvest scheduling.

A growing number of studies have highlighted the importance of managing uncertainty in driving agriculture dynamics and productivity. Mathematical programming applications to consider randomness in agricultural activity are constantly growing including stochastic programming. Linker (2021) claims that «stochastic approaches are suitable for developing risk-adverse management strategies which are conceptually much closer to farmers reasoning, and are therefore more likely to be of practical interest». Furthermore, stochastic programming models are widely used for agricultural planning problems. These models are useful and applicable to real-world cases and provide robust solutions. A range of agricultural applications using stochastic programming includes irrigation scheduling and harvest planning.

3.1 Agricultural water management

Water is an essential input for agricultural production, and water management is crucial to respond to the water requirements of different crops. Water management is challenged by various factors as socio-economic pressures (prices, changing economic conditions, etc) and climate change risks (rainfall, droughts, heat waves, etc).

Irrigation scheduling is a key component in agricultural water management schemes. It is a management decision taken under numerous uncertainties. Optimal water allocation is challenging as it is «generally a decision taken with uncertainty regarding seasonal crop needs (unknown yield, precipitation, and other environmental factors)» (Berbel and Expósito, 2021). In this context, optimizing water resource management is essential to ensure sustainable and productive agriculture. The decision criteria aim to offer optimized irrigation schedules and to maximize profit.

During the last decades, many studies applying stochastic programming to agricultural water management and planning problems have been conducted. Afshar et al., (1991) used a chance-constrained optimization model to study reservoir planning for irrigation district. The model considers the eventual interactions existing between design and operation parameters (capacity of reservoir, area of land to be developed and planted, etc.). The model provides «the optimum extent of the land development for irrigation, cropping pattern, reservoir and canal capacities, as well as the necessary linear decision rule operational parameters». The model also brings out the importance of incorporating the reservoir cost in the model.

Huang (1998) developed an inexact-stochastic water management model based on an inexact chance-constrained programming method. The solutions provided by the model can be useful in providing insight regarding tradeoffs between economic and environmental objectives.

Many other studies have used two-stage stochastic programming (TSP) to optimize water management in the agricultural sector. The TSP is suitable for the analysis of medium- to long-term planning problems in which scenario decomposition resolution is required. The advantage of this technique is the concept of recourse, which provides the ability to take corrective actions. Two-stage stochastic program models make an initial decision in the first-stage based on uncertain future events. Once these uncertainties are resolved, a recourse or corrective action is taken.

Guo et al., (2009) proposed a two-stage fuzzy chance-constrained programming to optimize the management of water resources under dual uncertainties. This approach integrates a TSP and a fuzzy chance-constrained programming within a general framework. The model allows the analysis of many policy scenarios and provides reasonable solutions that help obtain the desired water allocation patterns.

Huang et al., (2010) suggest a simulation-based optimization method for planning water resources management systems under uncertainty. This method allows incorporating uncertainties expressed as probability density functions and discrete intervals into the optimization framework. The obtained solutions are reasonable and can be useful in generating adequate policies for supporting water management with maximized gains and minimized system-failure risk.

Lu et al., (2016) presented «a credibility-based chance-constrained optimization model for integrated agricultural and water resources management». Authors attempted to address parameter uncertainty by using the concept of credibility. The model represents uncertainties as fuzzy sets and provides a credibility level that indicates the confidence level of the obtained optimal management strategies.

Zhang et al., (2020) proposed a multi-objective chance-constrained programming approach for planning problems with uncertain weights. This approach would help to optimize economic profits and ecological benefits of the agricultural system over the planning horizon. The main contribution of this method is that it tackles uncertain objective weights and random parameters. To demonstrate the applicability of their model, the authors applied it to a case study of agricultural water management in northwest China. Results indicate that the proposed method provides robust solutions.

Zhang et al., (2021) formulated in their paper entitled «Irrigation water resources management under uncertainty: An interval nonlinear double-sided fuzzy chance-constrained programming approach» an approach that optimizes irrigation water allocation under complexity and uncertainty. The proposed approach combines double-sided fuzzy chance-constrained programming and inexact quadratic programming within a general optimization framework. It provides solutions that are useful for a better management of irrigation water in irrigated agricultural areas.

3.2 Optimization tools for harvesting

Harvesting is an important and complex operation for producers. It requires route planning that is subject to changes related to spatial crop yield and scheduling with support vehicles. Inefficient routing leads to negative impacts as increased labor cost, increased operation time and increased risk of final product quality degradation. It negatively affects the crop yield, hence the profitability for producers.

Timely and efficient harvest processes are crucial to preserve crop yield and quality, and minimize labor and machine maintenance costs.

Z. Jiao et al., (2005) focused on their paper on optimizing the harvest schedule to maximize gain in sugar content of cane. The harvesting process is better carried out when the likely sugar yields are at the season's peak. Unfortunately, this is not possible due to the limited capacities of harvesting. The authors used a statistical model and a linear programming model to obtain the best harvest scheduling, and hence to maximize the total sugar content in the sugar canes in a harvest season. The developed technique was applied to a real case, and the results showed potential gains in profitability.

M. Varas et al., (2020) proposed a model that seeks to maximize the quality of the harvested grapes and minimize the total operational costs of the harvest operation while considering several operational constraints such as the number of workers or machinery. To efficiently organize the process of harvesting, the authors formulated a multi-objective mixed-integer linear programming model and developed a negotiation protocol that can help decision-makers find a final harvest schedule.

Elbio L. Avanzini et al., (2021) considered operations planning to organize the process of harvesting grapes under uncertainty in weather conditions. Weather uncertainty affects the quality of grapes and thus their economic value. It is modeled following a Markov Chain approach. In this work, the authors compared an expected value with a multistage stochastic optimization programming for grapes harvesting planning operations. Results of the study indicate that results provided by the multi-stage approach are better than those provided by the expected value approach, especially under high uncertainty and high grape quality scenarios.

P.He et al., (2021) suggest optimizing harvesting and transportation simultaneously to reduce the total operational costs. The study focuses on wheat harvesting and transportation in fragmental farmlands.

First, the wheat should be harvested during the harvest season, and then the grains need to be transferred to the depot. The process brings out the interest and the importance of synchronization between harvesting and transportation. A joint optimization framework is employed to optimize jointly wheat harvesting and transportation problem. It is composed of vehicle routing problems with multiple trips and assignment problems. Furthermore, an effective hybrid algorithm is used to solve this complex problem and find an optimal solution. The results indicate that the model and algorithm are an effective approach to help farmers reduce operational costs.

4 CONCLUSION

Stochastic optimization deals with a class of optimization models that involve significant uncertainty. It belongs to the major approaches for dealing with uncertain parameters in optimization problems.

Data uncertainty abounds in many real-world optimization models. In the agricultural field, randomness is prevalent due to many parameters. For example, investment decisions in agriculture are implemented before many factors, such as weather, can be observed and before the demand for agricultural products is known. Such uncertainties further amplify the complexity of problems related to this field. Stochastic programming methods have been widely used to deal with uncertainties inherent to the conduct of agricultural operations and to provide assistance in decision-making. For instance, they can be used to optimize farmland management, irrigation scheduling, harvesting, and many other agriculture-related operations.

Stochastic optimization helps deal with the uncertainty inherent to agricultural operations and generates optimal management strategies to carry out these operations.

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