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3D Transformation Matrix Synthesis in Column Major and Row Major Forms: Applicative Perspective for 3D Object Representation

ShwetaChaku[1], Monika Sainger[2], Amrita Bhatnagar[3], RaginiKarwayun[4], SoumiGhosh[5]

[1]*Computer Science and Engineering Department,Ghaziabad,UttarPradesh, India*
[2]*Information and Technology Department,Ghaziabad,UttarPradesh, India*
[3]*Computer Science and Engineering Department, Ghaziabad,UttarPradesh, India*
[4]*Information and Technology Department, Ghaziabad, Uttar Pradesh, India*
[5]*Computer Science and Engineering Department, Ghaziabad, Uttar Pradesh, India*

Abstract.

The Transformation of a 2D or 3D objects are the effective means of shifting or changing the dimensions and orientations of images in the most effective way. If we fail to transform the object in terms of displacement ,enlargement, orientation, we may land up in creating something that is distorted and processessing a distorted object is not acceptable in the various critical fields like Medical and Forensics. The usual practice of defining transformations is straight forward. The transformed object can be obtained by coupling original object with the transformation vectors. The main challenge is how to evaluate it. The usual practice is standard Column Vector form. The alternative Row Vector Form is also known approach but what matters is the sequence of operations that make these both approaches worth mentioning.While doing so our analysis on 3D content keeps our knowledge flawless and takes it a step further as far as Image Processing is concerned. Such analytical study is very vital since most of the content created, acquired, reproduced, and visualized in 3D needs to be mapped on to 3D. This paper describes the transformations(Translation, Scaling and Rotation) in the both Column and Row Vectar Approach. This paper aims in providing a clear sequence of calculations which differ in both approaches.

Keywords: 3D Transformations, Homogeneous Coordinates, Rotation, Scaling,Translation,Reflection.

1. Introduction

The shape Computer Graphics needs strong visualization of 2D and 3D objects. Consider a analogy of an architect who builds bridges. It becomes important for an architect to study bridges from different angles. i.e. Front View, Side View, Top View .All such Transformations require a complete analysis of displacement ,zooming in zooming out and orientations. So if such images are converted into numbers, the numbers can be stored and manipulated for detailed analysis. This manipulation could be done using mathematical functions more precisely Matrix Operations. All of these transformations can be efficiently and succinctly handled using some simple matrix representations What makes it more overwhelming is the calculations behind all those transformation. My paper aims to compare the mathematical techniques laid down for such transformations and at the same time explores the distinct sequence of calculations by this comparison

1.1 Classification of 3D Transformations

There are basically five transformations which are listed as below:

- 3D Translation
- 3D Scaling
- 3D Rotation
- 3D Reflection
- 3D Shearing

1.2 Objectives

The Objectives for laying this analytical methodology is important as far as transformations of 3D objects are concerned .They aim at

- Developing a clear semantics in laying down the mathematical formulae and
- Laying the proper sequencing of operation while performing the concatenation of the transformations

1.3 Scope of the work

This work helps in analyzing the underlying formulae and comparison in two approaches towards transforming a 3D object.This work will be helpful to the researchers that study orientation of 3D objects and help them to analyze the calculation essentials required for **Translation ,Scaling and Rotation of 3D objects**.

2. Methodology

The methods in 3D Transformations involve two different approaches:

- **Column Vector Approach:** In the Approach we multiply Column Vector Transformation matrix with the Original Object matrix to get the Transformed Object Matrix
- **Row Vector Approach:** In this Approach we multiply the Original Object Matrix with the Row Vector Transformation Matrix to get the Original Object Matrix.

2.1 Column Vector Approach

. 3D Translation:

It can be defined as displacement or shift of a 3D image by some vector along x-axis and y-axis and collectively along z-axis

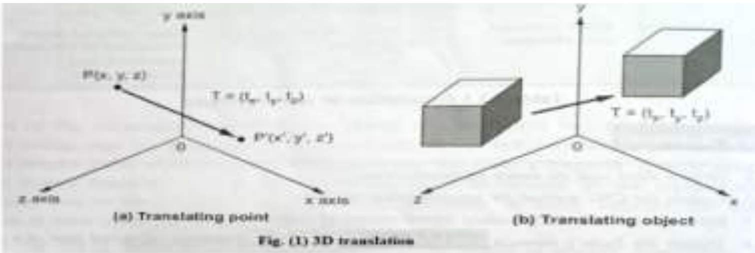

Fig. 1.Translation of 3D object.

We take a coordinate point as a starting point. What we need to show is the displacement of a original coordinate point to the translated coordinate point. Mathematically it can be formulated as shift vector along x-axis and shift vector along y-axis followed by the shift vector along z-axis thus transforming the point from P1(x,y,z) to P2(x',y',z').Thus translation is defined by adding a shift vector in the x y and z direction. These shift vectors can be termed as tx, ty and tz (They are also called the translation vectors):

Mathematically $x_2 = x_1 + tx$; $y_2 = y_1 + ty$; $z_2 = z_1 + tz$ (Translation is Additive)

Other transformation matrices like (Rotation,Reflection) take different form so to get all matrices in a common form.For this we introduce a homogenous system which means adding a dummy variable in 3X3 matrix to make it 4X4 matrix

Usual and preferred approach:

The Preferred Approach is the 4x4 matrix for 3D Translation which has the translation vectors in the third column. The matrix can be shown as below

$$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Translation Matrix (Column Form)

Consider Point P having the coordinates as (x,y,z) and T as 3D Translation Matrix. The Translated Coordinate P' (x',y',z') will be calculated as shown below

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{T} \cdot \mathbf{P}$$

3D Translation Equation in column vector form

Translated point with a column vector can be calculated by multiplying the translation matrix with the original coordinate.

AlternativeApproach: Translation in row vector form:

The Row Vector form takes the translation vectors in the fourth row and the sequence of operation to get the transformed point take the reverse order. Original Point coordinate has to be multiplied with translation matrix to get the transformed point. In this case the translated point P' will take the form

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$\therefore \quad P' = P \cdot T$$

3D Translation Matrix(Row Form)

Consider Point P having the coordinates as (x,y,z) and T as 3D Translation Matrix.The Translated Coordinate P' (x',y',z') will be calculated as shown below

$$[x' \ \ y' \ \ z' \ \ 1] = [x \ \ y \ \ z \ \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

3D Translation Equation in Row Vector form

Table 1.Evaluation on 3D Translation :

TRANSLATION	**Original Point**	**Matrix Form (Translation T)**	**Order for P'**	**Translation Equation**
Column-Major Form	P	$\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$	P'=P.T	$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

Row-Major Form	P	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$	P'=T.P	$[x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$

3D Scaling

Scaling is defined as the process of changing the size of an object in the x,y and z direction.(Resizing can change shape also).Scaling with respect to origin can be done by multiplying the coordinate values P1 (x,y,z) of each vertex of a object by scaling factors sx,sy,sz respectively to produce coordinate P2(x',y,z''). In this process, we can either enlarge or shrink the object. Such a scaling is called Pure scaling. Value of scaling factors can effect our object in a greater way for e.g; if sx=sy=sz>1, it means

Uniform Extention. If sx=sy=sz<1 means Uniform Reduction and if sx is not equal to sy which in turn is not equal to sz, it means Shape Distortion

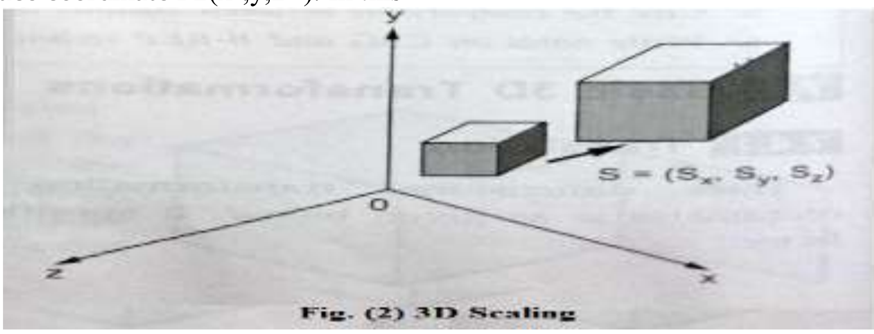

Fig. (2) 3D Scaling

Fig. 2.Scaling of 3D object.

The relation is: $x_2 = x_1 \cdot sx$ and $y_2 = y_1 \cdot sy$ and $z_2 = z1 \cdot sz$ (Scaling is Multiplicative)

Usual and preferred approach: scaling in column vector form

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Scaling Matrix(Column Form)

Point P1 has the coordinates as (x,y,z) and T is the Scaling Matrix ,then the transformed point P2 (x',y',z') will take the form.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Scaling Equation in Column Vector form

Thus,P2=T.P1

A Transformed point with a column vector can be calculated by multiplying the scaling matrix with the original coordinate.

Alternative Approach: Scaling in row vector form

The Scaling matrix takes the same form both in Column Vector Approach as well as in Row Vector Approach .

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix Notationfor 3D Scaling Matrix (Row Form same as Column Form)

We can confirm it by transposing the Scaling Matrix discussed in Usual and prefered approach.We get the same scaling matrix.The transformed point P' in a row vector form can be calculated by multiplying the original coordinate with thescaling matrix.The resultant matrix will be same as the resultant matri in column major form

Its important to mention here that if sequence of operation is vice versed then the resultant matrix will be incorrect.

$$P' = P \cdot S$$

$$[x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Scaling Equation in Row Vector form

Thus P'=P.S

where Point P has the coordinates as (x,y,z) and Point,S is the scaled matrix. P' will have the scaled coordinates calculated as (x',y',z')

Table 2:Evaluation on 3D Scaling :

SCALING	Original Point	Matrix Form (Scaling S Same in both)	Order for P'	ScalingEquation
Column-Major Form	P	$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	P'=P.S	$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$
Row-Major Form	P	$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	P'=S.P	$P' = P \cdot S$ $[x' \ y' \ z' \ 1] = [x \ y \ z \ 1] \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3D Rotation

The angle of the rotation along the axis of the location need to be specified in 3D rotation. 3D rotation is performed about X, Y, and Z axes.

Usual and preferred approach: rotation in column vector form

The matrix representation of 3D rotation is as follows :

$$x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\theta) & sin(\theta) & 0 \\ 0 & -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$y = \begin{bmatrix} cos(\theta) & 0 & -sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ sin(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$z = \begin{bmatrix} cos(\theta) & sin(\theta) & 0 & 0 \\ -sin(\theta) & cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D rotation Matrix about X,Yand Z

Let the angle of the rotation along the axis of the location be a. 3D rotation is performed about X, Y, and Z axes. Consider Point P1 has the coordinates as (x,y, z) and R be the Rotation Matrix then the Transformed point P2(x' ,y' ,z') will take the form

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos a & \sin a & 0 \\ 0 & -\sin a & \cos a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D rotation is performed about X-axis in column major

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & \sin a & 0 & 0 \\ -\sin a & \cos a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D rotation is performed about Z-axis in column major

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & 0 & -\sin a & 0 \\ 0 & 1 & 0 & 0 \\ \sin a & 0 & \cos a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D rotation is performed about Y-axis in column major

Thus P2=R·P1

Alternative Approach:Rotation in row vector

It is observed that in the Row Major form the Matrix of rotation remain same for

$$[x'\ y'\ z'\ 1] = [x\ y\ z\ 1]\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) & 0 \\ 0 & \sin(a) & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3D rotation is performed about X-axis in row major

$$[x'\ y'\ z'\ 1] = [x\ y\ z\ 1]\begin{pmatrix} \cos(a) & 0 & \sin(a) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(a) & 0 & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3D rotation is performed about Y-axis in row major

$$[x'\ y'\ z'\ 1] = [x\ y\ z\ 1]\begin{pmatrix} \cos(a) & -\sin(a) & 0 & 0 \\ \sin(a) & \cos(a) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3D rotation is performed about Z-axis in row major

Table 3:Evaluation on 3D Rotation :

ROTATION		Original Point	Matrix Form (Rotation)	Order for P'	Rotation Equation(Sign changes depending on direction of rotation)
Column-Major Form		P	$x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\theta) & sin(\theta) & 0 \\ 0 & -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $y = \begin{bmatrix} cos(\theta) & 0 & -sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ sin(\theta) & 0 & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	P'=R.P	$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos a & \sin a & 0 \\ 0 & -\sin a & \cos a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ $\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & 0 & -\sin a & 0 \\ 0 & 1 & 0 & 0 \\ \sin a & 0 & \cos a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

			$z = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$		$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos a & \sin a & 0 & 0 \\ -\sin a & \cos a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$
Row-Major Form		P	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) & 0 \\ 0 & \sin(a) & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} \cos(a) & 0 & \sin(a) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(a) & 0 & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} \cos(a) & -\sin(a) & 0 & 0 \\ \sin(a) & \cos(a) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	P'=P. R	$[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(a) & -\sin(a) & 0 \\ 0 & \sin(a) & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ **3D rotation is performed about X-axis in row major** $[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \begin{pmatrix} \cos(a) & 0 & \sin(a) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(a) & 0 & \cos(a) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ **3D rotation is performed about Y-axis in row major** $[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \begin{pmatrix} \cos(a) & -\sin(a) & 0 & 0 \\ \sin(a) & \cos(a) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

3 Results and Discussion

We can set up a matrix for any number or sequence of transformations as a composite transformation matrix by calculating the matrix product for the individual transformations. It is often referred as concatenation or composition of matrices.In concatenation of transformations , the sequence of the transformations play a vital role .This sequence is written from Right to Left(R->L)in column Approach of synthesis.But in Row Approach the sequence takes the order from Left to Right(L->R). [T1]*[T2] is not equal to the [T2]*[T1].

4 Conclusions

It can be concluded that if as an graphics analyst we don't draw the clear comparisons between the both approaches we might compromise on the issues of having simple ,consistent matrix notation using Homogenous Coordinate System and finally land up in misinterpretation of window modeling. Thus it is important to have a clear understanding between the sequencing of operations in both the approaches .Concatenation of Transformations multiplied with the Original Object in Column Vector Approach willequate same with Row Vector Approach only if Original Object is multiplied with Concatenation of Transformations

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