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Leading teachers who promote math learning

Dra. Luisa Morales-Maure [a, b], Orlando García-Marimón [b], Evelyn García-Vázquez [c], Marcos Campos-Nava [d], Jaime Gutiérrez [b], Miguel Ángel Esbrí [d]

[a] *SNI-I Member, Panama,*
[b] *Professor of Mathematics Department, Universidad de Panamá, Panama.*
[c] *Professor of Education's Faculty, Universidad de Panamá, Panama*
[d] *Professor of Academic Area of Mathematics and Physics, UAEH, México*

Abstract

The theoretical references that underpinned a proposal for teacher training of large-scale math teachers are presented, through a diploma course aimed at teachers in primary education service, oriented in the mathematical competences for teaching, of the Republic of Panama. The theoretical framework built for this purpose, is based on various theoretical perspectives and theoretical constructs, framed in the discipline of mathematical education; on the one hand, the importance of the use of conjectures in the class of mathematics was considered to justify the solution of problems during class; on the other hand, perspectives emanating from academic leadership that should be encouraged in the training of teachers in service were considered. In addition to the above, the theoretical perspective known as the OntoSemiotic Approach (OSA) was considered, which was the one that allowed to analyze the teaching activity of the participating teachers. The results obtained from the analysis, which are briefly mentioned in this work, suggest that the accompaniment to math teachers, by teachers with a high mastery of experts, allows their teaching empowerment to conform as academic leaders in their workplaces.

Keywords: empowerment, educational leader, diploma course, mathematics, teacher.

1. Introduction

One of the functions that a teacher should have is his/her ability to promote in his students an analysis of problems raised in class and not a simple reproduction of notions presents in used textbooks. When doing a thorough analysis of the problem (problem solving) present the teacher must ask questions that get his students to build ideas that can form a conjecture (National Council of Teachers of Mathematics, 2000).

Conjecture action is a null or almost absent activity in the classroom, which, however, when used systematically, can contribute to problem solving. Mathematics teachers must include conjecture within their resources, since in the mathematical community theorems are born of proposed conjectures and in that sense Brousseau (1986) argues that in mathematical work: [...] *the demonstrations obtained are rarely from the conjectures considered,* so every time new mathematical problems are *imposed*, the teacher should not solve it only as if the students were not in the *classroom (unguessed).* It is desirable that our students participate in the solution of some problem raised through an activity where these also: [...] formulate, *test, build models, languages, concepts, theories, that exchange them with others* (Lakatos, 1978).

The construction of mathematical thought in students could only be achieved if there is a real compression of how to do it and this can only be achieved by taking steps similar to when a professional mathematician demonstrates theorems, considering his class a scientific micro society (Brousseau, 1986). Where questions are asked and self-answered when students have no possible arguments, so that they are driven to present ideas that lead them to possible solutions.

Definitely not all guesses are good at solving a problem. Human beings are continually tried to answer or solve the problems presented to them in the daily life, but their guesses or answers are not always correct. It can be said that often these guesses lead him to create in him misperceptions of the problem that is intended to be solved.

With these series of activities, the teacher of mathematics encourages in his/her students a critical and scientific thought necessary to be useful in the society where they interact. However, often the teacher is not able to provide a respective analysis on the topics he/she "develops" in class due to an insufficient mastery experience, both at the disciplinary and teaching level. Then the question arises: How does the experienced math teacher achieve an expert mastery? It is important that the teacher himself has a solid mastery of discipline, and that he/she also has the ability to improve in various methodologies when there is the possibility of training in the didactics of the discipline, he/she teaches, and always be in constant "training" of the topics he raises in the classroom such as those that he finds difficult to develop.

Teacher training is a much-discussed topic in the educational community, as they need to have excellent training in their area. Therefore, they are reiterated that they are in a permanent training attending seminars, congresses, colloquies, etc. where there is a "true" exchange of experiences between teachers and with the researchers themselves that help facilitate their teaching work and improve their mastery.

In this order of ideas, Cantoral (1991) indicates that the teacher tends to examine mechanisms for building mathematical concepts and processes of calculation oriented by aspects of the physical thinking of prediction in phenomena of continuous flow in nature. This work shows how a concept is constructed in teachers and is often misinterpreted due to its low ability to engage it in other contexts. there is a tendency to consider more generic elements, in addition to responding to an excessively instrumentalized vision of them (Torres, et al., 2020, p. 296). Therefore, there is a need for teachers to improve with experts on specific topics that they must develop in their courses. The idea of teaching leadership has gained a lot of strength over the past decade. This is partly due to the notion that:

Teaching leadership aligns with the notions of individual empowerment and management localization that have spread throughout U.S. history. Specifically, the concept of teaching leadership suggests that teachers rightly and importantly have a central position in the way schools operate and in the core functions of teaching and learning. (York-Barr & Duke, 2004, p. 255)

According to Katzenmeyer & Mollers (1996), definition [teaching leaders] are those who "lead in and out of the classroom, influence others to improve educational practice, and identify and contribute to a community of teaching leaders" (p. 6). York-Barr & Duke (2004) defined the term teaching leader in a similar way because they believed that teaching leadership is "the process by which teachers, individually or collectively, influence their peers, principals, and other members of school communities to improve teaching and learning practices with the goal of increasing student learning and achievement" (p.287–288). This is not to say that teachers who are not leading teachers are not good and effective teachers, but that leading teacher have "something extra" that sets them apart from just being a good teacher. Curtis (2013) summed it up this way:

Given our newly refined ability to distinguish between teachers and their effectiveness, and the imperative imposed by common core standards to provide instruction on a more sophisticated level, it is no longer reasonable or sustainable to continue treating teachers in the same way. Instead, school systems must provide their highest-performing teachers with leadership roles that both elevate the profession and enable them to have the greatest impact on colleagues and students. (p. 3)

The concept of teacher leadership has received attention because schools and school districts are constantly working to develop strategies to improve schools and student learning (Franklin, 2012). Schools are realizing that the principal or administrative leader cannot face the challenge of improvement on its own, as today's demands are unprecedented (Curtis, 2013; Danielson, 2007). Most would agree that teachers with effective instructional strategies are essential for improvement to occur. The goal of teaching leadership is to improve student performance by reflecting the characteristics of effective classroom teachers (such as work ethic and personality) and transferring these critical skills to improve teacher pedagogy. York-Barr & Duke (2004) further stated that:

Recognition of teaching leadership comes in part from new knowledge of organizational development and leadership that suggest that active participation of people at all levels and within all domains of an organization is necessary for change to take hold (Ogawa & Bossert, 1995; Spillane, Halverson & Diamond, 2001). Educational improvement at the instructional level, for example, necessarily involves teacher leadership in classrooms and peers. (p.255)

When we are asked what kind of educational leadership skills a teacher needs, the answer is given by Esbrí et al. (2021): During the emergency, leaders and their teams must adopt a "test and learn" attitude, be prepared to recognize what works and what does not work in order to immediately adapt the process being applied in business schools and are very useful for teachers.

There is a disagreement about the importance of certain elements in student learning. Some believe that class size is an important component (Pianta & Hamre, 2009; Ponitz, Rimm-Kaufman, Brock & Nathanson, 2009; Castro et al., 2021; Torres, et al., 2022). Others believe that social factors play the most influential role (Cummins, 2012; Schofield & Bangs, 2006), while some claim that socioeconomic status is one of the most important variables (Gassama, 2012; Naiditch, 2010).

Regardless of which topic is most valuable, it would be difficult to find someone who disagrees with the notion that "No principle of school reform is more valid or lasting than the maxim that student learning depends first, last and always on the quality of teachers"(Usdan et al., 2001, p. 1).

In the same way Garcés, Font y Morales-Maure (2021) mentioned that:

Several studies on mathematics teaching in engineering degrees have indicated that the high number of students who fail basic science subjects in these degrees is directly related, among other aspects, to the way in which teachers' approach and teach mathematics [...] (p. 5)

In this order of ideas, Nava & Díaz (2019), propose to characterize the declarative CPC of the professors of Physics by means of categories or dimensions, which allowed them to develop a closed-scale Likert test, with which they could give and identify how developed the CPC of a physics teacher is on a specific topic.

2. Significance of the Study

The main objective of the Diploma Course: <<*Didactic Strategies for the Teaching of Mathematics* -DSTM>> (EDEM; in Spanish Estrategias Didácticas para la Enseñanza de las Matemáticas) is to create leaders in elementary mathematics that (a) would increase participants' content and pedagogical knowledge, and (b) increase the self-efficacy of math participants. The DSTM program was designed to train teachers, designated as SENACYT fellows by being accepted into the program, for four months, where the development of activities was in their classroom to be leaders in front of their students. The months of professional development included two parts: teachers were advised and trained by math specialists, and as they were exercise teachers they passed on their knowledge to students. The questions sought to be answered through the analysis of the participation of fellows in this formative process were:

1. Did teachers increase their knowledge of the content and their self-efficacy in mathematics?

2. How did participants perceive the professional development they received?

3. What challenges did teachers face in collaborating and training classroom students?

Ultimately, the researchers sought to determine how a year-long sustained professional development experience contributed to the growth of elementary school teachers in science and mathematics as teaching leaders. This research examined these questions regarding participants' perception of their experiences during training.

3. What is the DSTM program?

DSTM is a teacher training initiative based on the need for continuous education training through which a research-based teacher (diploma course) math support training program emphasized problem solving and hands-on learning from the Didactic-Mathematical Knowledge and Competencies (DMKC) theoretical model. Based on OntoSemiotic Approach (hereinafter OSA) constructs to perform an analysis of mathematical activity in the processes of teaching and learning mathematics. These sciences are dedicated to the study of the characteristics of educators and their students as well as the conventions, norms and organization of educational institutions (Pochulu, Font & Rodríguez, 2016; Hummes, Font & Breda, 2019; Vanegas, Font & Pino-Fan, 2019; Godino, Batanero & Font, 2020; Breda et al., 2021; García Marimón et al, 2021; Maure, Font, Gonzalez, & Vasquez, 2021).

Using the DMKC as the basis for training, teachers facilitated the implementation of effective professional development strategies. The study of mathematics enhances access to science and technology by imparting methodological rigor and encouraging the exercise of executive functions necessary for personal and professional life (Faragher, Stratford, & Clarke, 2017; Bottge, B. A., Cohen, A. S., & Choi, H. J., 2018). This suggests the need for the student to receive knowledge that supports growth and confidence in their own knowledge and judgment (Morales Maure, García Vazquez & Durán González, 2019). This situation contravenes the goal of the school as a socializing agent, since it affects its contribution to the development of mental functions and even more, to the integral education of the learner (García Marimón et al, 2021).

This bridged the gap between successful and innovative short-term practice and the long-term institutionalization of teaching strategies. These strategies promote teacher understanding and incorporation of research-based principles of science and math instruction to positively impact student performance. (Kuhlthau & Maniotes, 2010). When we refer to research, our perspective is based on the description of the National Research Council (1996):

Scientific research refers to the various ways in which scientists study the natural world and propose explanations based on the evidence derived from their work. Research also refers to the activities of students in which they develop knowledge and understanding of scientific ideas, as well as understanding how scientists study the natural world. (p.23)

This work suggests trends to follow for teaching mathematics, which can be inferred from relevant publications in the area, - for example: manuals (Handbooks) on research in Mathematics Education (Bishop, et al., 2003; Gutiérrez & Boero, 2006; English, Bartolini-Busi, Jones, Lesh, & Tirosh, 2008) or publications from a series of studies by the International Commission on the Teaching of Mathematics (ICMI) (Hanna, 1996; Mammana & Villani, 1998; Fauvel & Maanen, 2000; Holton, Artigue & Kirchgräber, 2001; Kendal & Stacey, 2004; Batanero, Burrill & Reading, 2011), the creation of topic study group, at conferences (e.g. TSG 4 New advances and trends in education at the top level of ICME 11), or in the publication of journals specifically related to this topic (such as the Journal of Mathematics Education Trends and Research). On the other hand, several authors in the field of Mathematical Education have consistently reflected on what are the new current perspectives in the teaching of mathematics (Müller, 2000; Guzmán, 2007; Font, 2008).

The trends in teaching mathematics that we have presented support the thesis that mathematical knowledge includes the competence to apply them to real-life non-mathematical situations. In some countries, this principle has been embodied in the design of competency-based curricula for Primary and Secondary Education. These curricula provide for mathematical competence, which is understood, in a similar way to the conception contained in the PISA 2003 report (Maddison Angus & Organization for Economic Co-operation and Development, 2003). This report understands the notion of competence as the ability to understand, judge, do, and use Mathematics in situations where they can play a role (Niss, 2003) and consider the following more specific mathematical competencies:

1) Think and reason. Ask questions characteristic of Mathematics ("Is there...?", "In that case, how many?", "How can I find...?"); know the types of answers Mathematics gives to those questions; differentiate between the types of statements (definitions, theorems, conjectures, hypotheses, examples, conditioned assertions); and understand and treat the breadth and limits of given mathematical concepts.

2) Argue. Know what mathematical demonstrations are and how they differ from other types of mathematical reasoning; follow and value the chaining of mathematical arguments of different types; have a heuristic sense ("What can or cannot happen and why?"); and create and capture mathematical arguments.

3) Communicate. This involves knowing how to express yourself in different ways, both orally and in writing, on topics of mathematical content and to understand the oral and written statements of third parties on these topics.

4) Build models. Structure the field or situation you want to model; translating reality into mathematical structures; interpret mathematical models in terms of "reality"; working with a mathematical model; validate the model; reflect, analyze, and criticize a model and its results; communicate opinions about the model and its results (including limitations of such results); and monitor and control the model construction process.

5) Formulate and solve problems. Represent, formulate, and define different types of mathematical problems (e.g., "pure", "applied", "open" and "closed"); and solving different types of mathematical problems in various ways.

6) Represent. Decode and encode, translate, interpret, and differentiate between the various forms of representation of mathematical situations and objects and the interrelationships between the various representations; select and switch between different forms of representation depending on the situation and purpose.

7) Employ operations and symbolic, formal, and technical language. Decode and interpret formal and symbolic language and understand its relationship to natural language; translate from natural language to symbolic/formal language; handle statements and expressions with symbols and formulas; use variables, solve equations, and perform calculations.

8) Use brackets and tools. Have knowledge and be able to use different supports and tools (including information technology tools) that can aid in mathematical activity; and know their limitations.

Within the framework of the OSA, it has been considered that this formulation of the term of competence must be developed to be operational, and for this it is necessary to carry out a characterization of competition through levels of development and indicators that allow its development and evaluation (Godino, Batanero & Font, 2007).

Godino, Batanero & Font (2007) argue that the OSA is "the formulation of an ontology of mathematical objects that takes into account the triple aspect of mathematics as a problem-solving activity, socially shared, as symbolic language and logically organized conceptual system" (p.4). The development of skills in the classroom requires, on the part of teachers, a proposal of tasks that allow their development progressively. The teacher should include a plan to demonstrate this progress and evaluate its development by considering the need to incorporate a number of elements related to knowledge about the development of cognitive skills for his students (Morales-Maure et al., 2018).

In the DMKC model, the two key competencies of the math teacher are considered the mathematical competency and the didactical analysis and intervention competency. These competencies are characterized by the ability to design, apply, and value learning sequences of their own and others. They are also characterized using teaching analysis techniques and suitability criteria (Breda, 2020; Burgos, Castillo, Beltrán-Pellicer & Godino, 2020), to establish planning, implementation, valuation, and proposals for improvement. This research has focused on the competence of analysis and teaching intervention.

The exercise of the teacher's profession in the area of mathematics is a complex practice that requires a combination of knowledge and skills, "it is not only important to know what mathematics teachers know but also how they know them and what they are able to mobilize for teaching" (Chapman, 2014, p. 295) and there is agreement that the teacher who teaches the math class must be competent in the observation of tasks , not only to evaluate them by skills but to develop their work in an appropriate way: "the development of professional skills are largely activated through thoughtful learning, which allows us to understand the complexity of educational processes" (Alsina, Planas & Calabuig, 2009, p. 256). Therefore, you should be competent and reflect on both epidemic and cognitive analyses and properly manage pedagogical purposes.

It has been suggested that the teacher carry out an in-depth analysis of the teaching situation in the classroom having a broad mastery of the subject covered, but she must also know about her mathematical history because she studies the evolution of ideas as D'Amore (2007) argues for him, teaching is influenced by the conceptions of teachers about the nature of scientific knowledge and its evolution. Therefore, within the continuous training and training of a teacher it is necessary to include mathematical history as well as the epistemology itself to understand it correctly.

D'Amore (2007) broad that knowing only Mathematics is not enough if one does not have the very meaning of the evolution of mathematical thinking mainly in order to have an adequate instrument for the assessment of classroom situations in particular of "epistemological obstacles". Epistemology in conjunction with History is then seen as necessary for teacher training as support tools to understand the obstacles students often face in the classroom. But what are epistemological obstacles?

Bachelar (2004) mentions about the epistemological obstacles that have often hit me the fact that science teachers even more than the others can, do not understand that it is not understood. This misunderstanding is because there are a variety of obstacles, and it is necessary to understand to help to jump those obstacles presented in the students. One of the reasons is the absence of previous knowledge linked to the compression of a set of new ideas very own about how and why a mathematical concept.

From this arises the importance of studying how knowledge was built throughout history. That is, the evolution of scientific thought through different civilizations and really know the importance of the History and Epistemology of Sciences in the training of the professor. In this respect, on the Epistemology of Sciences it must be clear that the knowledge that is handled today in the classroom was not always presented in this way, it is more known that there was a need to change the questions present in history to others by modifying the ideas that allowed the evolution of that scientific thinking.

In Mathematics Education, when talking specifically about mathematical thinking, the following question arises: How can the teacher then manage to create numerical, algebraic, geometric thoughts, among others which allow in them a development of science in the classroom? Here it is important to note that the teacher is the main proponent of mathematical skills aligned with the development of the fundamental knowledge of his students. In terms of school math skills, researchers distinguish it from mathematical creative skill such as that associated with the ability to solve problems and math tests where the ability to reason using symbols is also related and that the mathematical thinking process is related to the ability to draw conclusions, organize structures, and manipulate relationships (Castro, 2013). A teacher must develop these skills within his classroom both school and creative to drive alternative solutions to those posed in the classroom that ignite acquired "innate" skills.

These skills are necessary in the formation of an individual, which are affected by verbal obstacles. It is therefore important that the teacher has a broad culture that has an impact on his "argumentative discourse" within the classroom, where he is knowledgeable of a wide variety of topics that he includes within his "vocabulary" with other elements that are binding on school mathematics and serve as a bridge to facilitate his students' appropriation of the mathematical concepts studied.

All elements or resources mentioned so far should be included in the role of the teacher as an academic leader, but it is important to talk about didactics specific to the Didactics of Mathematics that studies the teaching-learning process of mathematics in the classroom. This relatively young discipline reflected through

Godino (2002) as one that generates a set of knowledge that serves to explain and inform the communication and acquisition of mathematical content, this set of knowledge is generated by a line of research followed by a group of specialists interested in it, who are responsible for building knowledge that meets the criteria marked by the rules of the scientific method.

Then didactics in Mathematics within its challenges tries to solve the problem in the teaching-learning process or at least give a "coherent" explanation to the phenomena that occur in the classroom. The educators and teaching teams are responsible for developing useful theoretical constructs to "optimize" this learning teaching process that allows to solve events as marked as the case of PISA tests in Panama and other Latin American countries where each year three reflects the low capacity that students have in mathematics compared to other participating countries.

In this sense, discipline specialists take basic questions for this discipline: - what to teach (mathematics), - why (philosophy), - to whom and where (sociology), - when and how (psychology). Godino (2002) citing Higginson's works aligns with the following topics such as: understanding traditional positions on teaching-learning mathematics; understanding the causes of curriculum changes in the past and forecasting future changes; and the change of conceptions about research and teacher preparation. The results showed significant student achievement, students who attended DSTM schools and classes for four months showed a gain in the assessment of math problem solving with the DMKC model compared to non-exercise participants of the Ministry of Education.

By way of closure, it can be observed that an academic leader must have a preparation that includes expertise in his discipline (disciplinary part), knowledge of the theoretical constructs typical of the learning teaching process (specialized didactic part) and the management of useful teaching resources for the benefit of his students. All the above results in an efficient education that allows its students to operate in society productively, making them critical and knowledgeable of the scientific aspects developed in the world.

Teaching leaders are needed for meaningful and sustainable change within schools, as they can be central to a school system's ability to improve student performance and teacher job satisfaction (Hanuscin, Carina & Somnath, 2012; Rhoton & McLean, 2008). Higher education institutions should partner with school systems to structure opportunities for future candidates to work under the tutelage of leading teachers. This will allow the trainees an opportunity for collaboration as they learn their craft and will also learn the additional benefits of becoming a leading teacher.

Teachers are necessarily at the center of the reform because they must meet the requirements that accompany the high standards in the classroom (Cuban, 1990). To meet these demands, "teachers must be immersed in the subjects they teach (content) and have the ability to communicate basic knowledge to develop advanced thinking and problem-solving skills (pedagogy) among their students" (Garet et al., 2001, p.916). However, while many teachers generally support high standards for teaching and learning, many are not prepared to implement teaching practices based on these standards (Cohen, 1990; Elmore & Burney, 1998).

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