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# Assessment of Year Five Pupil's Number Sense

Piriya Somasundram

*Faculty of Education, Languages, Psychology, and Music SEGi University, Malaysia, piriyasomasundram@segi.edu.my*

## Abstract

Algebra has been a major stumbling block for many middle and higher secondary school students. Vast research has shown that the abrupt introduction of algebra in middle school causes students to fail to connect the relationships between arithmetic learned in primary school and formal algebra in middle school. Thus, algebraic thinking is necessary for the transition from arithmetic to algebra. One of the elements of algebraic thinking is number sense. It allows students to make sense of numbers and operations rather than focusing solely on numeric calculation. Therefore, this study aimed to assess the number sense of year five pupils in one of the states of Malaysia. Quantitative analysis was performed on the performance levels of 720 year five students in five number sense tasks. The majority of the students didn't demonstrate number sense. They solved the tasks based on rules, which showed the effect of computation-centric learning. Nevertheless, there is no correlation between the ability to work with number sense and their mathematics achievement. This indicates that the ability to think algebraically does not correlate with achievement in school mathematics examinations. It could be interpreted that high achievers in school tests don't acquire a conceptual understanding of number sense. This sheds some light on how high achievers in primary school mathematics could result from rote learning and drilling, leading to a struggle in algebra. It is time to revisit the teaching of arithmetic in primary schools to develop conceptual understanding and sense-making of numbers to excel in algebra in later years of education.

**Keywords***:* Number Sense, Algebraic Thinking, Generalisation, Relational Thinking.

## 1. Introduction

In the landscape of mathematical proficiency, two pillars stand tall — algebraic thinking and number sense. These fundamental concepts form the bedrock of mathematical literacy, providing a comprehensive framework for understanding the relationships between numbers and the ability to manipulate them effectively. Together, algebraic thinking and number sense create a symbiotic relationship that not only empowers individuals with problem-solving skills but also fosters a deeper appreciation for the inherent beauty and logic within the world of mathematics.

Number sense is the intuitive grasp of numbers and their relationships, transcending mere memorization of facts. It involves a fluid understanding of quantities, the ability to estimate, and a keen awareness of the inherent structures within numerical systems. Number sense lays the foundation for mathematical fluency, enabling individuals to make informed decisions, solve real-world problems, and develop a genuine appreciation for the role of numbers in our daily lives.

Algebra plays a critical role in school mathematics and securing many career opportunities in STEM (Steele et al., 2016). Algebraic thinking is a crucial aspect of mathematics that involves understanding and analyzing patterns, relationships, and structures. However, Algebra has been a major obstacle for many middle and higher secondary school students (Kaput, 2008). Algebraic thinking from an early age is an important part of mastering formal algebra in middle and high schools (Wettergren, 2022). Some elements of arithmetic can be instilled to develop algebraic thinking. One of those elements is number sense (Adamuz-Povedano et al., 2021). Algebraic thinking, built upon the principles of number sense, takes mathematical reasoning to the next level. At its core, algebraic thinking involves the systematic manipulation of symbols and expressions to represent and solve mathematical problems. The introduction of variables and the exploration of patterns and relationships characterize this stage of mathematical development. Algebraic thinking is not merely a set of rules but a dynamic process that encourages creativity, critical thinking, and logical reasoning.

Number sense is known as an essential skill to deepen the understanding of the number system (Jordan et al., 2022). It emphasises the number and operations relationships. In addition, it develops the ability to perform mental calculations and numerical estimation. The mathematics lessons in primary school must interweave the making sense of numbers to develop a conceptual understanding of algebra from a young person. Lack of making sense of numbers leads to problems when the students encounter handling inversion and precedence of operations (van Amerom 2002). Thus, this study attempted to examine the year five pupil's performance level in the number sense and its correlation with mathematics achievement in school mathematics examinations.

Number sense and algebraic thinking complement each other in a symbiotic relationship. Number sense provides the foundation upon which algebraic thinking can flourish (Scalise & Ramani, 2021). When individuals possess a strong number sense, they are better equipped to identify patterns, make connections between mathematical concepts, and understand the significance of variables in algebraic expressions.

Conversely, engaging in algebraic thinking refines and deepens number sense. The abstraction inherent in algebraic manipulations encourages a more profound understanding of numerical relationships. This symbiosis is evident in how algebraic thinking not only refines arithmetic skills but also broadens the understanding of mathematical structures and operations.

## 2. REVIEW OF LITERATURE

### 2.1 Early Algebra

Teaching algebra early is different from early algebra. Early algebra contrasts with middle and high school algebra. It serves as a bridge connecting arithmetic and algebra, facilitating a gradual introduction to algebra starting from arithmetic. Algebra is a subset of arithmetic and representational systems. Early algebra refers to the introduction of algebraic concepts and reasoning in the early years of mathematics education, typically during elementary school. Rather than focusing solely on rote memorization of formulas, early algebra emphasizes the exploration of mathematical structures, patterns, and relationships. Students engage in activities that involve identifying and extending patterns, generalising, and expressing mathematical ideas symbolically. Early algebra serves as the scaffolding for the development of algebraic thinking. It introduces fundamental concepts that become the building blocks for more advanced algebraic reasoning later.

Research in cognitive development supports the integration of early algebra into elementary education (Vanbinst et al., 2018). The period between ages 5 and 11 is crucial for developing abstract reasoning skills. Early exposure to algebraic concepts aligns with the cognitive development milestones of students, making it an opportune time to lay the groundwork for more advanced mathematical thinking.

### 2.2 Algebraic Thinking

Numerous individuals might question what constitutes algebraic thinking and how it distinguishes itself from formal algebra. The literature has examined algebraic thinking from diverse viewpoints. As per Mason (1996), algebraic thinking involves recognizing similarities and differences, discerning variations, categorizing, and labeling, encompassing the search for algorithms. Kieran (1996) has described algebraic thinking as “the use of any of a variety of representations that manage quantitative situations relationally” (pp. 274-275). To elaborate on this, Blanton and Kaput (2003) highlighted that algebraic thinking also deals with representations and looking for relationships. Arithmetic thinking centres on familiar numbers and calculations, whereas algebraic thinking involves exploring relationships between numbers and the capacity to derive the general case. Kaput (2008) has classified algebraic thinking into two aspects, generalisation and symbolisation. Three strands further developed from these two aspects. They are generalised arithmetic, modelling, and function. Various perspectives of algebraic thinking fall into one of these strands.

Generalised arithmetic is “the study of structures and systems abstracted from computations and relations, including those arising in arithmetic (algebra as generalised arithmetic) and quantitative reasoning” (Kaput, 2008, p. 11). Generalised arithmetic fosters thinking algebraically while solving arithmetic problems. When students formulate a broad statement encompassing numerous instances, they engage in arithmetic generalization (Kaput et al., 2008). For example, recognizing that the sum of two odd numbers always results in an even number constitutes a generalization about addition. Primary school mathematics lessons must incorporate skills to identify generalise patterns. An example of this is that appending one to an even number consistently yields an odd number. If a number is divisible into two equal groups, it is necessarily an even number.

Kaput defined modelling as “the application of a cluster of modeling languages both inside and outside of mathematics” (p. 11). Commonly, the elementary school syllabus comprises some elements of algebra. For instance, questions about finding missing addend or minuend in a simple word problem; a simple mathematical sentence that looks like " 20 + __ = 24", " __ + 5 = 27", and " 54 - __ = 33". Also, these concepts extended to multiplication and division. Such as "8 × __ = 56", and " 54 ÷ __ = 9". These questions which demand to find the unknowns are algebraic and provide the opportunity for the students to grasp the properties and relationships between arithmetic operations (Carraher & Schliemann, 2007).

Kaput (2008) described functions as “the study of functions, relations, and joint variation” (p. 11). Proficiency in handling patterns might suggest the early development of algebraic thinking in children. For example, the identification of patterns in various scenarios and the utilization of symbols and variables to represent patterns and generalizations are crucial components of early algebraic thinking. Using patterns is seen as a way of approaching algebra (Mason, 1996).

As discussed above, algebraic thinking can be developed at an early age while teaching arithmetic. These skills evolved to develop essential skills for thinking algebraically. The children must be exposed to algebraic thinking in primary school which could enable them to learn informal algebra and eventually prepare to master formal algebra in secondary school. For instance, understanding equal sign as it is not a sign which requires computing but rather shows equality, would enable them to work with complex linear equations in the middle and high schools. By then they will have already been familiar with the generalisation and able to conceptualise the algebraic form. This skill would prepare them to work with variables and unknowns without attempting to compute and find the answer immediately. With these insights in hand, students will find that algebra is not a mystery, but a territory that already has familiar landmarks. Thus, the following section discusses the importance of number sense in developing algebraic thinking in primary schools.

### 2.3 Number Sense

In general, the primary mathematics curriculum addresses real numbers, commonly referred to as arithmetic. Frequently, arithmetic is considered a foundational step before delving into algebra. Nevertheless, the structure of the school mathematics curriculum tends to treat arithmetic and algebra as separate and distinct subjects (Cai & Moyer, 2008; Herscovics & Linchevski, 1994). Hence, students commonly encounter difficulties in bridging the cognitive divide between arithmetical and algebraic concepts. (Herscovics & Linchevski, 1994). In arithmetic, only straightforward calculations are involved with known numbers (van Amerom, 2003). For instance, 3 + 5 = 8, which means that nothing more, nothing less. Particularly, working from known to unknown using computations is traditional arithmetic. Meanwhile, reasoning about the unknown when it proceeds from the unknown, via the known, to the equations is formal algebra. Hence, the difference between arithmetic and algebra is that the former involves a specific situation while the latter involves a general solution (van Amerom, 2003).

Number sense is the fundamental ability to understand and manipulate numbers, providing the foundation for more advanced mathematical skills. It encompasses an intuitive grasp of quantities, relationships between numbers, and their relative magnitudes, serving as a precursor to mathematical fluency. It is crucial in daily life to make informed decisions related to budgeting, shopping, and other practical tasks. Individuals with a strong number sense can estimate costs, compare prices, and manage finances more effectively, contributing to better financial literacy. Developing number sense enhances critical thinking and problem-solving skills. It involves the ability to analyze and reason numerical information, helping individuals approach mathematical problems with confidence and creativity. A strong number sense fosters mathematical confidence. When individuals have an intuitive understanding of numbers, they are more likely to engage with and enjoy mathematical concepts, leading to increased self-efficacy in learning and applying mathematical principles. Number sense is not plainly important in basic arithmetic; it is a steppingstone for more advanced mathematical concepts. Proficiency in the number sense facilitates understanding algebraic relationships, statistical analyses, and complex mathematical operations, enabling success in higher-level mathematics.

According to Hsu, Yang, and Li (2001) number sense can be divided into five components. These involve comprehending the meanings and relationships of numbers, acknowledging the scale of numerical values, grasping the relative impact of operations on numbers, formulating computational strategies, assessing their logical validity, and possessing the capability to depict numbers through various representations.

## 3. METHODOLOGY

### 3.1 Objectives and Research Questions

Knowing the importance of number sense in fostering algebraic thinking, this study aimed to investigate year five pupils' ability to make sense of numbers and its correlation with mathematics achievement in a district of Malacca. The objectives of this study are twofold as follows.

1. To investigate the year five pupils' performance level in number sense tasks in the district of Malacca.

2. To investigate if there is a relationship between year five pupils' performance level in number sense tasks and mathematics achievement.

In line with these objectives, the following research questions were formulated:

1. What is the year five pupils' performance level in number sense tasks in the district of Malacca?

2. Is there any significant relationship between year five pupils' performance level in number sense tasks and mathematics achievement?

The data in this research originated from fifth-grade students attending a National school within a district in Malacca. Schools were randomly chosen using the list provided by the Ministry of Education for this study. Upon selecting a school, the study encompasses all fifth-grade students within that specific school. The samples are 720 year-five pupils which comprised 370 females (51.4%) and 350 males (48.6%).

Participants were tasked with completing these five assignments using paper-and-pencil assessments. The dichotomous scoring method involved assigning a score of 1 for correct responses and 0 for incorrect or blank responses. Subsequently, the gathered data underwent analysis using SPSS (version 28.0) software. Data analysis involved the application of both descriptive and inferential statistical techniques.

Table 1 presents the mid-year mathematics examination grades of the sampled schools. In the mid-year school examination, mathematics was passed by 83.3% of the samples. Approximately 66.3% of them achieved satisfactory and moderate results, earning grades A, B, and C. Overall, the fifth-grade pupils involved in the study demonstrated a commendable performance in their school mathematics examinations.

Table 1 Samples' Mid-Year Mathematics Examination Grades.

| Grade | Frequency | Percentage |
|---|---|---|
| A | 118 | 16.4 |
| B | 156 | 21.7 |
| C | 203 | 28.2 |
| D | 122 | 16.9 |
| E | 120 | 16.7 |
| Missing | 1 | 0.10 |
| Total | 720 | 100.0 |

## 4. DATA ANALYSIS

The results reported here were part of a major study's results. Table 2 displays the total number of correct responses for the five tasks. For Task 1, 13.5% of year five pupils were able to make a reasonable estimation for the division without computation. In Task 2, 40.0% of the samples were able to find the greater fraction without attempting to make a common denominator. The number sentence in the third task comprised estimating the greatest output based on the multiplier. About 70.3% of the samples were able to choose the number sentence with the greatest results. The fourth task is a little easier and involves the estimation of the product. The samples were required to find the product of two-digit numbers without performing multiplication. As this task involved two-digit numbers which are closer to 20, samples managed to make an educated guess by estimating the closest to 400. Therefore, 59.9% of the samples were able to answer this task. Lastly, about 71.9% of the samples were able to find the estimated position of the number given on a number line. However, it is presumed that they did grasp the underlying concept of a number line.

As anticipated, most of the samples were unable to perform well in Task 1 better compared to other tasks. Overall, the year five pupils' performance was moderate in estimation and making sense of numbers. A majority of the samples successfully identified the unknown values presented in the number sentences for both Task 1 and Task 2. The samples encountered difficulty in determining unknowns during Task 3 and Task 4, which entailed non-canonical representations of number sentences and the inclusion of two unknowns.

Table 2 Number of Correct Responses for the Five Number Sense Tasks

| | Frequency | Percentage |
|---|---|---|
| Task 1 | 97 | 13.5 |
| Task 2 | 288 | 40.0 |
| Task 3 | 506 | 70.3 |
| Task 4 | 431 | 59.9 |
| Task 5 | 518 | 71.9 |

The Pearson product-moment correlation coefficient was employed to investigate the correlation between the ability in number sense and achievement in mathematics. The requirement of the number sense test is the variables should be ratio or interval measurements. Thus, the scores of these five tasks were converted to percentages. After that, the relationship between the performance in number sense tasks and mathematics achievement was investigated. The researcher analysed the correlation between scores obtained from number sense tasks and mathematics marks from their school midyear examination. The outcome showed there is no correlation between number sense tasks and mathematics achievement (p>.05). Thus, It indicates that there is no correlation between the scores of number sense tasks and mathematics achievement.

Table 3 displays the results from Pearson product-moment correlation test using SPSS software.

Table 3 Pearson Product-Moment Correlation Coefficient

| | | MidYear | NumberSense |
|---|---|---|---|
| MidYear | Pearson Correlation | 1 | .044 |
| | Sig. (2-tailed) | | .237 |
| | N | 720 | 720 |
| NumberSense | Pearson Correlation | .044 | 1 |
| | Sig. (2-tailed) | .237 | |
| | N | 720 | 720 |

## 5. DISCUSSION & CONCLUSION

As indicated by the results of the five tasks (see Table 2), the majority were unable to estimate 8.12 ÷ 10 in Task 1 and they could not relate the connection with the decimal places and zeroes. The sample was able to perform well in Tasks 3 and 5. This could be for two reasons. Firstly, Task 3 was solved using computation rather than making sense of the multiplier. Secondly, the pupils are able to work with the basic number line. On the other hand, most of the participants were not able to depict the magnitude of fractions by looking at the denominators (Task 2). This ability was an indication of a lack of algebraic thinking (Blanton & Kaput, 2003). The percentage declined slightly for Task 4 because it involved multiplication and estimating the product. The participants could have found the right answer by computation. Regardless of the method, more than half of the participants demonstrated established the ability to make an estimate.

The outcomes of this study might offer more suggestive insights than conclusive findings, given the restricted number of tasks included in the current investigation. However, the results discussed in the previous section showed that year five pupils can make a reasonable estimation based on the number of sentences given without computation. As this is part of a major study, the further detailed data collection on how they obtained the specific answer shows that the students estimated the answer without numerical calculation. It's worth mentioning that these five number sense tasks may not fully unveil the participants' depth of algebraic thinking capability. Nevertheless, it can serve as a strong indication of the participant's ability to comprehend numbers in the early years. The extensive sample size in this study offers a comprehensive perspective.

Recognizing the interconnectedness of algebraic thinking and number sense, educators play a pivotal role in nurturing these skills from the early stages of learning. Integrating hands-on activities, problem-solving tasks, and real-world applications into the curriculum enhances both number sense and algebraic thinking. Emphasizing the relevance of these concepts fosters a positive attitude towards mathematics and empowers learners to see the beauty in the intricate dance between numbers and symbols.

The findings of this study shed some light on the year five pupils' capability to work with numbers without abstract computation. It also revealed that their ability is lacking when required to work without rule-based and make a reasonable estimation. It can be concluded that the year five pupils who participated in this study acquired only a surface level of understanding when working with numbers. Therefore, it is time for curriculum developers to pay attention to the importance of instilling sense-making of numbers in earlier grades and incorporating it into the arithmetic syllabus.

In the journey toward mathematical proficiency, the synergy between algebraic thinking and number sense is undeniable. Together, they form a dynamic duo that not only equips individuals with problem-solving prowess but also instills a genuine appreciation for the elegance and order inherent in the realm of mathematics. As we continue to unlock the potential of these foundational concepts, we pave the way for a future generation adept at navigating the complexities of an ever-evolving world with mathematical fluency and confidence.

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