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An Analysis of the Co-Constructed Learning Process among Mathematics Grounding Activity-Designers

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Abstract

The "Just Do Math (JDM)" project, initiated in 2014, aimed to build the ground that would promote students' learning mathematical interest and achievement and cultivate "Mathematics Grounding Activity (MGA)-designers" and "MGA-teachers" for designing and implementing MGA modules. Consequently, this study aimed to explore module designers' role perceptions and its developmental processes in the JDM Project. An exploratory qualitative approach was employed to reach the objectives. Data were gathered through observations, interviews, and various kinds of documents, and then qualitatively analyzed by the editing and immersion analytic techniques. Findings were reported as followings: First, MTEs claimed that the four-element PD model (i.e. goals, contexts, theories, and structure) was employed in the JDM project for conceptualizing the MGA-designers' PD programs. Grounded on this argument, MGA-designers' PD process was correspondingly analyzed and portrayed consistent with the four elements: Goals—Building grounds by doing mathematics and solving problems; Theories—Emerging into design-based PD process; Structure—Co-constructed learning and symmetrical roles of MTEs and MGA-designers; Context—Analogous and interactive learning process. Secondly, within the whole co-constructed and designed-based PD activities, MGA-designers endeavored to simultaneously learn how to design and exercise the design task of MGA modules. Thus, MGA-designer A's designing process was used to illustrate these MGA-designers' learning process within the PD program, where two contrary cases were presented to compare and contrast his designing processes. In these two cases, the three-phase "problem-solving" model (i.e. entry, attack, and review) was employed for describing the learning context, where the four-element PD model is embedded correspondingly.

Keywords— Mathematics Grounding Activity Designer, Co-Constructed Learning, Professional Development

Introduction

• JDM Focused on Establishing the Ground of Students' Ability and Continuous Teachers' PD

According to the findings of PISA 2012, the mathematical performance of Taiwanese 15-year-old students ranks fourth in all participating countries. However, the gap between high-achieved and low-achieved students is enormous (i.e. up to 245 points), which equals to the difference that obtaining 6-year education may have (PISA in Taiwan, 2015). Among seven top-scored Asian countries, both the proportion and quantity of disadvantaged groups in Taiwan is also the largest. These issues reveal that students' mathematical achievement is polarized and appears to have M-shape phenomenon. Therefore, the "Just Do Math (JDM)" project, initiated by the Ministry of Education in 2014, aimed to build the ground that would promote students' learning interest and achievement in mathematics as well as cultivate "Mathematics Grounding Activity (MGA)-designers" and "MGA-teachers" for designing and implementing MGA modules. Eventually, it hopes that every student can learn math successfully (Lin, 2014).

From "No Child Left Behind" to "Every Child Success Act", proposed by federal government of the United States, the educational policy focuses more on cooperative learning at all educational levels. Improving the quality of teaching and learning in the backward 5% schools for the purpose of enhancing their students' basic abilities and skills stands at the center of this reform movement. It also emphasizes the preparation of high-quality "literacy" and "STEM (mathematics particularly)" teachers by continuously advancing better professional development program (both pre-service and in-service), which may fulfill the abovementioned goal. Similar to this American reform focus, the vision of the JDM project in Taiwan is to view mathematical learning as the ground of acquiring important life-related knowledge and skills that all children are able to apply what they learn to their future lives. Moreover, it values the establishment of children's basic ability; that is, furnishing them to the "base line" (i.e. mathematic maturity baseline), especially for those who with low-readiness or low-achievement. In fact, for the sake of achieving these objectives, providing high-quality teachers' professional development programs where student-centered mathematics activities can be designed is the main task of this project.

- **JDM is a Multi-Wins PD Platform for Educators, Teachers (MGA-Designers), and Students**

Furnishing in-service teachers with adequate opportunities of professional development (PD) stands at the main task of the teacher education system in every country (Garet, Porter, Desimone, Birman, & Yoon, 2001; Sandholtz, 2002). Can (2011) indicates that in-service training can not only enhance teachers' knowledge, experience, and skills but also help them to understand their own responsibilities of continuous professional development in the future. A successful in-service teacher training program focuses on evaluating students' academic grades (Vo & Nyugen, 2010), improving teachers' quality (Goldschmidt & Phelps, 2010), and making school policies better (Day, 1999). In fact, giving teachers more autonomy on designing and implementing their teaching activities recently gets greater attentions in Taiwan while discussing how to evaluate teachers' professional knowledge and abilities. Based on our curriculum standards, teachers have the right to choose different versions of textbooks and adjust how to teach the content flexibly (Ministry of Education, Taiwan, 2011). In addition, teachers are asked to both fully understand the content/concept(s) before deigning and implement it and authentically perceive students' possible learning problems or difficulties in order to arrange appropriate learning activities with productive struggling (Schoenfeld, 2013). Providing applicable teacher professional development programs, where teachers are able to interact with experts and peers and learn how to apply theories into practices, may arouse their teaching passion and, in turn, achieve the abovementioned goal.

Chung, Lu, and Shih (2007) indicate that most of teachers in the central advisory group in Taiwan didn't perceive their own roles and tasks. Later on, they further claim that more studies are needed to clarify their roles and tasks for future improvements (Lu and Chung, 2010). In fact, within the JDM project, these teachers in the central advisory group, served as MGA-designers, are in charge of both designing MGA modules and training future MGA-teachers. Therefore, it is important that how these MGA-designers' roles and task perceptions were developed within the professional development program. Besides, with regard to the design of the professional development program, there are various studies of successfully adapting diverse theories to individually differentiated professional development models (e.g. using different strategies while training) in the field of mathematics in different countries (Becker & Pence, 1996; Lin, 2000; Loborde,1999; Tirosh

& Stavy, 1999; Tsai, 2004). Collectively, recent studies emphasized applying the "co-constructing" framework, i.e. mathematics teacher educators (MTE) and teachers, to design the mathematical professional development program, including work delimitation and task design (Chin, 2014; Lin, 2014; Jaworski, 2008；Zaslavsky, 2008). Within this framework, MTEs and mathematics teachers, with much closed relationship, simultaneously are all learners who reflect and construct mathematical teaching tasks and practice them together in order to bring the benefit to all students. At the same time, every teacher may have different recognitions and interpretations on various theories, issues, and viewpoints (Jaworski, 2008). Moreover, Jaworski (2008) reveals that these teachers' knowledge and tasks, under the framework of the social-cultural theory, can be considered as a process of learning accommodations, where MTEs and mathematics teachers still possess their own uniqueness during the whole PD process (even though they are culturally different). Grounded on this argument, since the PD design has a significant impact on all participants' (i.e. including MTEs, and mathematics teachers), the developmental process of the PD program deserves to be carefully studied. Similarly, the PD program of the JDM project applies this "co-constructing" framework while designing and implementing the PD activities for MGA-designers. Therefore, it is essential to evaluate the professional development process of these MGA-designers for further explorations and future improvements. Consequently, two main questions of this study were: What were the essential elements of MGA-designers' PD process? What was the learning process of MGA-designers?

Theoretical Framework

- **21st Century Skills for Mathematical Learning**

"How can mathematics education prepare students for being able to participate in the digital society of the future?" (Gravemeijer, Stephan, Julie, Lin, & Ohtani, 2017, p. 106), this is definitely a critical question for all educators in mathematics education. To answer this question, we need to focus on what students learn in this fast-changing world. Several organizations and educators claim that students must develop 21st century skills, which are originated from the 21st Century Skills Project (Partnership for 21st century skills, 2015), to be effective post-education (Gravemeijer, et al., 2017). In fact, mastery of key subjects, include English, reading or language arts, world languages, arts, mathematics, economics, science, geography, history, government and civics, and 21st century themes is essential to student success. In addition, schools must promote an understanding of academic content at much higher levels by intertwining the five "21st century interdisciplinary themes" into key subjects, i.e. global awareness; financial, economic, business and entrepreneurial literacy; civic literacy; health literacy; environmental literacy (Partnership for 21st century skills, 2015). Within this framework, three categories of essential skills are required: learning and innovation skills, information, media, and technology skills, and life and career skills. Skills of creativity and innovation, critical thinking and problem solving, communication, and collaboration are prepared for our students to confront their increasingly complex life and work environments in today's world. Moreover, Voogt and Pareja (2010) argue that a list of similar skills are considered as general 21st century skills that include critical thinking and problem solving, collaboration across networks, agility and adaptability, initiative and entrepreneurialism, effective communication, accessing and analyzing information, and curiosity and imagination.

Regarding mathematics education, it is also importance to assist our students' mathematical learning for their future in the digital society. Gravemeijer, et al. (2017) propose their views on what mathematics education should prepare students for applying mathematics in all kinds of workplace and daily life situations, where they primarily emphasize the use of mathematics with an eye on employ-ability in the following three different perspectives: (1) The characteristics of mathematics at the workplace are outlined to provide a whole picture of what mathematical activity students have to be prepared for. (2) The mathematical competencies that complement the work of computers are identified for anticipating the demands of a computerized environment in the 21st century. (3) The mathematical content is verified to conjecture on how the increasing employment of information technology influences the mathematical topics that get more attentions under the effects of the use of information technology (Gravemeijer, et al., 2017). As they remind us, "choosing for 21st century skills and high-level conceptual understanding requires a significant effort in teacher professionalization, curriculum design, and test design" (Gravemeijer, et al., 2017, p. 120). Teacher professional development and curriculum design are key factors of successfully furnishing our students with these 21st century skills.

- **Co-Constructed Learning**

Zaslavsky (2008) propose "the dynamic nature of task design and implementation for teacher education" structure, which compiles several opinions and tasks practiced between mathematics teachers (MTs) and mathematics teacher educators (MTEs). Under this structure, by providing well-designed mathematical tasks, MTEs effectively assist MTs to develop their own potential and promote MTs to reconstruct their mathematical content knowledge and practical teaching experiences. The role of MTEs is "promoter" where their knowledge backgrounds and experiences are influential to the structure (e.g. tasks designed for MTs' learning) and they grow up simultaneously through the process of designing and implementing the designated learning tasks. Therefore, both MTEs and MTs are beneficial in this dynamic co-constructed learning process. Later on, Javaski (2008) claims that teaching mathematics is a complicated task, where teachers need to understand the content (e.g. mathematics concepts) and teaching strategies and then merge them into the teaching and learning process accompanying the use of proper assessment tools. It is also essential that teachers need to be familiar with the characteristics of students and the school context. Within the practical teaching process, there are still a lot of elaborate elements that teachers must be aware of. Accordingly, MTEs and MTs work collaboratively on developing curriculum and instruction are beneficial for students' mathematical learning. In this collaboration, MTEs are able to furnish MTs with sufficient content and pedagogical knowledge through various professional development activities that are grounded on their own professional knowledge and capabilities, which will, in turn, help MTs to learn and reflect during practical implementations in real settings. If MTs can apply what they learn, e.g. mathematics concepts and materials, mathematical learning theories, empirical evidences drawn from previous studies, and contextual teaching strategies that are supplied by MTEs, into their practical teaching process, the "co-learning" will be authentically initiated. Moreover, Javaski (2008) indicates that relationship between MTEs and MTs share some relevance in their knowledge, where they have consistent purpose and the same as learners. Also, since MTEs are not omniscient, they are still learners while helping MTs to practice their tasks.

Chen, Lin, and Yang (2018) argue that students, teachers, and teacher educators are all learners within the professional development program and its implementation process. Similar to Zaslavsky's (2008) viewpoint, the Lighten-Up School-Based Program (LUSBP) was initiated to plan innovative teaching themes in mathematics (e.g. mathematical conjecturing, diagnostic teaching, and reading comprehension), where MTs implemented the designated teaching theme in their classrooms with the assistance of PD workshops provided by MTE (Lin et al. 2011). In this program, a "design-based PD" is employed as the main framework in which MTE is not only "to design and implement mathematical tasks" but also "to integrate related theories for helping teachers in their design of innovative teaching activities" (Chen, Lin, & Yang, 2018, p. 518). Consequently, MTs' roles are "symmetrical" to MTE's: MTE is the designer of PD workshops, the educator of MTs, and the reflective practitioner of mentoring practice, while MTs are the learners within the PD workshops, the designers and teachers of the teaching activities, and the reflective practitioners of mentoring students' learning process. Within this co-constructed learning environment, the approach MTE designs PD activities changed "from being based on the literature content toward being learner-centered activities with teachers as learners", which "not only enhanced teachers' learning outcomes, but also facilitated the MTE's own professional growth in different areas, including mathematics, mathematics learning, mathematics teaching, teacher education, and, in particular, the extrapolation of generic examples for understanding mathematical concepts" (Chen, Lin, & Yang, 2018, p. 517). That is, this co-learning process is authentically beneficial for MTE and MTs. This PD closely connects teaching practice and theory, no matter MTE or MTs are able to enhance their own capabilities. Grounded on the design of LUSBP, the PD process in this JDM project employs the co-constructed learning approach (Chen, Lin, & Yang, 2018) and the dynamic nature (Javaski, 2008; Zaslavsky, 2008) in the task design and implementation process, where MTEs and MGA-designers are able to learn collaboratively in this co-constructed structure. Therefore, analyzing the implementation of the JDM PD programs will help us to understand the learning process of MGA-designers for future improvements.

- **JDM's PD—Four-element Model**

Loucks-Horsley et al. (1998) present a framework to guide the design of "professional development for teachers of mathematics (MPD)", which features the ways programs are designed to specific contexts and goals, as well as guided by knowledge and beliefs of teaching and learning. These three elements, i.e. contexts, goals, and knowledge and beliefs will affect the plan that is implemented and then evaluated for further revision. Grounded on this argument, Sztajn, Campbell, and Yoon (2011) propose a definition of a model for MPD, which are composed of four elements: goals, contexts, theories, and structure. In this definition, the structure of a MPD intervention is what mathematics teachers experience as participants. Goals define what is to be achieved through specific learning tasks. Sowder (2007) claims that the goals of a MPD may include:

"(a) a shared vision for mathematics teaching and learning, (b) a sound understanding of mathematics for the level taught, (c) an understanding of how students learn mathematics, (d) a deep pedagogical content knowledge, (e) an understanding of the role of equity in school mathematics, and (f) a sense of self as a mathematics teacher" (Sztajn, Campbell, & Yoon, 2011, p. 87).

Contexts refer to two important aspects, i.e. curricular and ambient context, that conceptualize the designated goals and shape the learning environment.

Employing appropriate theories about teaching and learning provides essential guidelines in framing a MPD (Borasi & Fonzi, 2002; Sowder, 2007). "Theory of teacher change" and" theory of instruction", proposed by Wayne et al. (2008), are both included in this model. "The more congruency between these two sets of theories, the more effective an MPD is likely to be" (Sztajn, Campbell, & Yoon, 2011, p. 88). Structure stands at the center of the model, which "is shaped by and un-detachable from the goals, contexts, and theories" (Sztajn, Campbell, & Yoon, 2011, p. 87) that guide the design of all learning tasks. Both content and format are necessary for framing the structure of a MPD. Content may comprise "the mathematics topics covered, a focus on student learning of particular topics, or a focus on mathematics curriculum", while format characterizes "how opportunities for learning are organized and presented (e.g. number of contact hours, span, location, type of contact, the activities carried out, and the artifacts used)" (Sztajn, Campbell, & Yoon, 2011, p. 89).

In fact, Lin, Yang, and Wang (2016) employ this four-element model for conceptualizing the professional development programs of the JDM project, where the goal is to help students engage in mathematics learning cognitively and affectively and the theories concerned two dimensions: student learning and teacher learning. Moreover, ground on the co-constructed learning structure, MTEs and MFA-designers share symmetrical roles in the JDM PD program: MTEs design PD workshops, guide and educate MGA-designers, and reflectively monitor/evaluate MGA-designers' learning process, while MGA-designers learn within the PD workshops, design and implement the MGA modules, and reflectively monitor/assess students' learning process. Therefore, because of the analogous nature of MTEs' and MGA-designers' roles, this four-element model (i.e. goals, contexts, theories, and structure) is applied into the design of MGA-designers' PD, where they learn how to design MGA modules and actually execute the designing tasks within this collaboratively co-constructed PD activities.

- **Doing Mathematics in the Real World—Design-based Problem Solving Phases**

As Gravemeijer, et al. (2017) argue, it is important that embedding 21st century skills (e.g. skills of creativity and innovation, critical thinking and problem solving, communication, and collaboration) and high-level conceptual understanding in designing mathematics teachers' PD as well as mathematics curriculum and instruction. As mentioned above, MGA-designers learn how to design and initiate the design tasks of MGA modules simultaneously, where they need to use the aforementioned skills to practically solve possible problems they may face during the design process. In order to design an appropriate MGA module for cultivating students' essential skills, MGA-designers have to personally experience this thinking and problem-solving and work collaboratively in this design-based PD process (Lin, Yang, & Wang, 2016). This paralleled learning experience is favorable for furnishing MGA-designers with authentic opportunities of "doing mathematics in the real world, which echoes to the goals of the JDM project.

Regarding the process of mathematical problem solving, several scholars propose various viewpoints. Polya (1945) first categorizes the problem-solving process in four stages: Understanding the problem, devising a plan, carrying out the plan, and looking back. Later on, Mason, Burton, and Stacey (1982) propose a three-phase model of mathematical problem solving, i.e. entry, attack, and review. At the first "entry" phase, one encounters the problem itself and starts to think about how to solve the problem by asking questions of "What do I know? What do I want? What do I need to link the two?". At the following "attack" phase, one endeavors to resolve the problem by following the thinking started in the entry phase and convincing itself and others. At the last phase "review", one tries to take a step back and begins to analyze the effectiveness of previous phases by checking, reflecting, and extending to a wider class of problems. Based on this argument, this three-phase "problem solving" model is employed in the MGA-designers' PD process of JDM project. Therefore, the learning process of MGA-designers will be presented corresponding to this model.

Research Design

- **Methodology and Participants**

An exploratory qualitative approach (Creswell, 2014) is employed to reach the objectives. This study was conducted during the 2016~2017 academic year, where there were eight one-day MGA-designers' PD classes. Participants were 3 MTEs (#1~3) and 6 MGA-designers (#A~F) who actively engaged in the whole process of the JDM MGA-designers' PD program. MTEs were Prof. Fu-Lai Lin (MTE1) and another two university professors who worked closely to the JDM projects starting from the beginning stage (in the year of 2014) of this project. MGA-designers were senior mathematics teachers (MTs) of the national/central advisory group (i.e.

mathematics education field, grade 1 to 12), who perform intensively as the first group of the participants of the JDM project. In fact, these MTs were first trained as MGA-teachers at the beginning stage of the JDM project, and then, in turn, intensively engaged in the process of designing the first set of MGA modules. Later on, they, as qualified MGA-teachers, were also responsible for hosting fun-math camps for verifying the effectiveness of the first set of MGA modules and revising those modules if needed. After taking part in the project three years (from 2014 to 2016), they voluntarily participated in the MGA-designers' PD program.

- **Data Collection and Analysis**

Based on the purpose of this study, data were gathered through non-participant observations, in-depth and follow-up interviews, and various kinds of documents (e.g. drafts of MGA modules, reflections, designers' learning portfolios) collected during the whole PD process. The data gathered were first categorized and pre-analyzed by five steps (Thomas, 2000): preparation of raw data files, closed reading of text, creation of categories, overlapping coding and uncoded text, continuing revision and refinement of category system. Both editing and immersion analytic techniques were used for further analyses (Crabtree & Miller, 1999). Through applying the organizing code topics mentioned in the theoretical framework section, the editing analytic system focused on MGA-designers' PD in the whole learning process. Because of the exploratory character of this study, the immersion analytic system was employed to explore essential information for the module-designing tasks within the PD program, in which the cycle of immersion was repeated until the described interpretation was reached (Crabtree & Miller, 1999). Within this analytical process, the replication logic of the data is applied associated with the triangulation techniques for generating and verifying the findings.

Findings

- **Essential Elements of MGA-designers' PD Process**

Lin, Yang, and Wang (2016) claim that the four-element model, i.e. goals, contexts, theories, and structure raised by Sztajn, Campbell, and Yoon (2011), is employed in the JDM project for conceptualizing the PD programs. Grounded on this argument, MGA-designers' PD process is briefly portrayed consistent with the four elements.

- Goals—Building grounds by doing mathematics and solving problems

The main goal of the JDM project is to help students engage in doing mathematics in the real world through the problem-solving process. Therefore, how to develop the targeted MGA-designers' comprehension of the designated goal and their capability in designing adequate MGA modules is the first task. Since most MGA-designers engaged in hosting fun-math camps and previous designing workshops of MGA modules, MTE1 believed that they were familiar with the rationale and theories of the JDM project. Accordingly, he encouraged them with "Baduanjin (Eight-Section Brocade)" of Yi Jin Jing of Shaolin Temple in the introduction of the first period:

"I understood that it was not easy to apply appropriate theories into the design process of MGAs; like me (with my age), I spent so many years to accumulate a hug mass of experiences that could help me to apply them into practices. However, I truly believed that these theories were practically applicable while conducting task analyses in designing MGAs." (OB-082416)

MTE1 also remarked three key principles in designing MGA modules,

"First, you needed to choose proper learning materials that matched students' previous life-related experiences [entry]. Moreover, the activity designs of MGA modules must be meaningful, which referred to what mathematical concept(s) students were going to learn and what ground(s) were you going to build [attack]. Finally, since you wanted your students to authentically possess those capabilities established in the learning process, you needed to include a reflection task in your final review worksheet, where they could seriously think about what they have done during the whole learning process [review]." (OB-082416)

These principles echoed with the three phases, i.e. entry, attack, and review, raised by Mason, Burton, and Stacey (1982).

In fact, after understanding the designated goal and starting to design the MGA modules, MGA-designer C reflected her own MGA design:

"My original purpose was to design a module for the language of "times (e.g. how many times, double, triple…)". Based on my previous teaching experience, I usually followed the content of the textbook in teaching this concept. In today's discussion, I found that it was too fast to teach the formal concept, which didn't match students' understanding or thinking. With regard to this "times" concept, they normally reflect from the knowledge of adding two or more numbers together; rarely, they own a comparative notion between two numbers." (IN-101416)

After her presentation and discussion, she learned various opinions from MTEs and other MGA-designers. She further found that, grounded on the evidences of previous studies, this issue may result from the limitation of children's language development, which is still a debatable issue. Consequently, she decided not to limit herself on this "times" language issue and think deeply about "how could I solve my students' learning problems" (IN-101416). She said, "in learning addition and/or substation, students may say 'five dollars more or less' and are able to express this idea reciprocally. However, they may not achieve this level in learning multiplication and/or division" (IN-101416). During the process of presentations and discussions in these PD classes, MGA-designers C learned an important structure while teaching mathematical operations, where children may be limited by their language development and teachers have to think about how to solve this potential problem. Actually, MGA-designer C said, "I will design more operation-based tasks for students to concretely manipulate teaching aids or life-related objects" (IN-101416). Consequently, it is evident that MGA-designer C perceives the parallel role of MGA-designers, where they are also learners in this designing process. Through the process of personally doing mathematics themselves, they are able to disenthrall their original teaching models and begin to re-think about how to design an proper curriculum along with employing multiple instructional strategies. This reflection shows that this kind of interactive "co-constructed" PD activities is authentically useful for MGA-designers to continuously develop their teaching and problem-solving strategies, which are beneficial for them to "disenthrall themselves from their previous-owned mathematical thinking" and "literally learning by doing" (IN-101416).

- Theories—Emerging into design-based PD process

Except the introduction of "Baduanjin (Eight-Section Brocade)" in the first class, MTE1 asked all MGA-designers to analyze whether the module "Factor Adventure" conforms to the design principles of MGA modules. He referred to the use of the teaching aid "Cuisenaire Rods" in this module (OB-082416), which echoed the four principles and stages of mathematics teaching and learning of Dienes (1973) based on structuralism. In this module, the context of "children's joyfully learning through manipulating those teaching aids embodies the contribution of play-based learning theories, where they experience meaningful learning through the process of free-play", said MTE1 (OB-082416). Besides, examples of understanding children's cognitive development and thinking, knowing correspondent teaching strategies and learning theories, and perceiving MGA modules' design principles are also embedded to demonstrate how to properly design MGA modules (RR-

082416). Through this real task-design example, MTEs addressed the importance of connecting theories with the design of MGA modules within this design-based PD process. Besides, at the first interview, MTE2 noted the reason of MGA-designers' PD, which he mentioned that MTE1 has worked with the central and local advisory groups over a long period of time. During this interactive process, he found that our students spent less time on thinking while solving mathematical problems, which may have a negative impact on their understandings of the designated mathematics concepts. Usually, students instinctively write down the answers right after they have a quick look on the questions. MTE1thinks that the most important elements of thinking and problem solving are missing in learning mathematics, which is the main reason he initiates this JDM project that can truly make up the learning gap for those students with low readiness. He said,

"Our students were used to wait for answers and didn't really want to think about how to solve the problems. I thought it might result from the instructional strategies, where teachers normally lectured the mathematics concept(s) and then directly got into calculation(s)." (OB-082416)

This teaching mode only provides students opportunities for technically practicing how to compute and obtain the answer instead of meaningfully comprehend how to solve the problem. In fact, MTE1 believes that learning mathematics shall be interesting and students need to make sense of what they learn; for example, "they could get better understandings through the operation of teaching aids or concrete objects, or pose questions while thinking how to solve the problem" (OB-082416). However, they lack this kind of meaningful learning experiences in classrooms.

- Structure—Co-constructed learning and symmetrical roles of MTEs and MGA-designers

Within the 2016~2017 academic year, there were eight one-day MGA-designers' PD classes. These eight PD classes emphasize actual designing practices, which are appraisingly criticized by MTEs and MGA-designers. At the first class, MTE1 introduced the core rationale of the design of MGAs—"Baduanjin (Eight-Section Brocade)" and criticized some MGA modules. At the second class, MGA-designers shared their reasons why they participated in this PD program as well as their preliminary thoughts and understandings of MGAs. Afterward, they worked in small groups to choose certain MGA modules for further revisions, which aimed to prepare themselves in thinking and designing new MGA modules later. During the third class, some MGA-designers presented their first draft of newly designed MGA modules and tried to answer questions or concerns raised by MTEs and other MGA-designers. More newly designed MGA modules were presented at the fourth class with discussions. In the meantime, the targeted MGA-designers endeavored to listen to each other, share opinions, ask questions, discuss how to design, where they collaboratively learn from these presenting processes in this co-constructed learning process. They perceived that they needed to disenthrall themselves from teaching mathematics only with an emphasis on lecturing, defining, and calculating. Instead, they became to re-think how to learn mathematics in a "student-centered" ground, which focused more on "doing mathematics".

During the last four classes (5^{th} to 8^{th}), MGA-designers kept presenting their revisions of the newly designed MGA modules, which were revised based on previous discussions and suggestions. These MGA modules were mostly modified at least three times. A series of task-based actions in thinking, convincing, and communicating occurred among MTEs and participating MGA-designers in this revision process.

- Context—Analogous and interactive learning process

Within the whole co-constructed and designed-based PD activities, MGA-designers endeavor to simultaneously learn how to design and exercise the design task of MGA modules. MGA-designer A's designing process is used to illustrate these MGA-designers' learning process within the PD program, where two contrary cases are presented to compare and contrast his designing processes. In these two cases, the three-phase "problem-solving" model (Mason, Burton, & Stacey, 1982) is employed for describing the learning context, where the four-element MPD model is embedded correspondingly.

- **A MGA-designer's Learning Process**
- Case I—Entry

Regarding to the goal of designing the module "throwing darts", MGA-designer A recalled the main reason of choosing "mean" as the core concept: "Mean" is one of the statistical concepts listed in our standards and included in the textbooks as well. I remembered, in one workshop or meeting, MTE1 mentioned that this concept (and other statistical concepts) hasn't been designed yet" (IN-082416). Based on this defect, MGA-designer A decided to initiate the design process of this module [goals], starting from reviewing relevant information of the standards and textbooks.

- Case I—Attack

After examining the content related to "mean" in the standards and the textbooks, MGA-designer A started to design the module "throwing darts" and presented his first draft of the module in the third class. Three activities were included in this module, one main activity and two extended activities. In the main activity, students are grouped (four students per team) for a dart competition. Every member in a group can throw five darts and record the scores. At the end of this game, they need to calculate the total scores. However, before this calculation, students have to discuss how to use a "representative number" to portray a "group score", which is contributed by every group member (i.e. scores of throwing 20 times of darts totally). During the discussion, the MGA-teacher will guide students to reach a common consensus of employing "mean" as the representative number. Then, students will calculate the mean scores for their groups for further comparison, where they can decide which team is the winning team. With regard to the two extended activities, pokers are used for determine how many times (darts) one student in each group can throw or decide how to conduct the later comparison. However, the two extended activates are "too complicated to understand for our students" (OB-093016), said MGA-designer B in MGA-designer A's presentation [structure]. MGA-designer D paralleled MGA-designer B's concern, "I think the two extended activities don't necessarily have to use the concept of mean; I think students will randomly choose a representative number, which may not make any sense of it" (OB-093016) [contexts]. MGA-designer E also raised his concern about how to select a representative number. He said, "Why do they have to use 'mean'? They may choose 'mode'; especially when they see certain scores are repeated several times" (OB-093016). Since several MGA-designers addressed similar issues of the two extended activities, it promoted MGA-designer A to rethink about his design.

In the fifth class, MGA-designer A exhibits his second draft, where the main activity was the same as the first draft but the rest were even more complicated. After his presentation, MTE1 proposed a question while MGA-designer A presented his module: [theories/contexts]

"For example, both teams have thrown nine times. For both teams, they face a critical choice on picking up one student to throw the tenth dart to win. Which student are they going to choose: a student who is stably scored but not outstanding or another student who score variously (sometimes high but sometimes low)?"

This choice can be made based on two different situations. If your team is currently the winning team, you may want to choose one player with stable scores before. If your team falls behind, you may want to take a risk and send a player who is probably able to get a higher score (occasionally but not stable). These two players actually represent two factors for this kind of game (i.e. throwing darts): one is accurate and the other is stable. For the "accurate" factor, "measure of central tendency" shall be included; for instance, "mean" is a determinant number, which is the core concept of your design. In fact, "measures of dispersion" is used for the "stable" factor, where "variance or range" can be employed as a determinant number. However, this "stable" factor is not included in your MGA. Of course, if one player who simultaneously owns these two features, i.e. accurate and stable, s/he will be the best player for any situation in that game. If only one feature is considered, it will become the same choice that MTE1 mentioned above, "a student who is stably scored but not outstanding or another student who score variously (sometimes high but sometimes low)". "This is the main problem of your design since only one factor is taught/considered in your game", said MTE1 (OB-102816).

In this game, for instance, two teams may obtain the same average score (e.g. M=20 points). Even though their means are the same, the variances of their original scores are probably different; for instance, one team is dispersed and another team is centralized. In this situation, these two "means" represent different meanings because another factor "measures of dispersion" is not included in this MGA module [theories/contexts]. In other words, the two factors, i.e. "measure of central tendency (e.g. mean, mode, or mode)" and "measures of dispersion (e.g. variance or range)", together shall be integrated to explain what the statistical meaning of the data is and how to make decisions according to the data (RR-102816). In the abovementioned game, how to choose the last player is so critical that may affect the final result (i.e. win or lose). As MTE1 reminded,

"Only one factor is used in your game. Students are not able to make a proper decision to win the game. Even if they just use single factor to decide who is going to be the last player, it is not necessary to use "mean" for this decision; probably, they may use 'mode' to figure out the factor 'stable' and choose the right player. In this case, when are they going to use 'mean'?" (OB-102816)

The opinion and suggestion of MTE1 made MGA-designer A to think about the main issue of his task design [theories/contexts/structure].

MGA-designer A: The "stable" factor…, but their scores…

MTE1: Right, for this player, his scores are stable but not accurate. So, his scores are not high enough. Another player's mean score is higher but not stable; sometime high but sometimes low. Which one are you going to choose for the last player? In fact, you have to consider the two factors I mentioned to make the decision.

MGA-designer A: Yes, in this case…

MTE1: Ok, this is what I talked about…how to select an appropriate player. So, your module has to be re-designed to include both factors.

[MGA-designer A is thinking; no further response]

MGA-designer B: Or, he can just play with the first factor, and then…

MTE1: If only one factor (i.e. accurate) is included, "mean, mode, and median" needs to be considered together.

MGA-designer B: I see. For example, if they have the same "mean", they have to consider another factor "variance". So, this case needs more in-depth discussions.

MTE1: If the three numbers of the "measure of central tendency" are taught together, students will further understand the statistical meanings of them. This will be a better design.

(OB-102816)

- Case I—Review

After these discussions, MGA-designer A conducted a pilot teaching with only four students, while they were divided into two pairs for this dart competition. However, this pilot teaching was not successful, where two similar issues existed and were addressed in the presentation and discussion of the sixth class. Since only two students per group, the decision of "choosing the last player" discussed above in this competition was made based on a very limited data set, which was neither theoretically nor practically correspondent to the basic principles of using statistical concepts. In fact, students didn't follow the rule of using "mean" as the representative number to choose the last player; "sometimes the one with the highest score (e.g. 100 points), the 'mode' (i.e. the one with several '90 points' scores), or the total score" (OB-111816). Moreover, MGA-designer B mentioned that the extended activities were still "too complicated for these students, which drew a critical concern of the goal of this module—what is the "ground" and how to build it" (OB-111816) [goals]. The fact was that, said MGA-designer F, "it looked that these students had no idea why and how to make decisions during the discussion process" (OB-111816) [theories/contexts]. Besides, there was less opportunity for students to reflect on their decision-making processes during or after they played the game [structure]. MGA-designer D pointed out this issue after MGA-designer A's presentation (RR-111816) and raised the same thought as MTE1's concern. In short, if students failed to acquire the designated concept (i.e. mean) and employ it in the learning process of this module, this pilot teaching was unsuccessful.

In the aforementioned discussion process, MTE1 endeavored to use real-life related situations to inspire all MGA-designers (especially MGA-designer A) to think about the defect of this MGA module. In the real world, both "accurate" and "stable" are key factors in a "throwing darts" game [theories/contexts]. These two factors are originated from the two statistical concepts "measure of central tendency and measures of dispersion". For MGA-designers, possessing a full understanding of this statistical knowledge is essential for designing the designated MGA module. In this case, MGA-designers A finally gave up re-designing this module. In a well-designed MGA, students will be furnished with the designated mathematical concepts. Therefore, a probably legitimate reason is that MGA-designer A's content knowledge (i.e. statistical concepts) is not sufficient enough to support him to design a suitable activity that includes the two factors (RR-111816). Since only one factor is employed in this game, students may be confused about what concept(s) guide them to make sense of the data and, in turn, figure out how to make a proper decision to win the game. For MGA-designer A, this inadequate mathematics content knowledge (MCK) led to his limited mathematical thinking and incomplete problem solving, which, in turn, resulted in an unsuccessful MGA design.

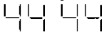
Table 1. Unsuccessful case—"throwing darts"

Problem-Solving Phases	Parallel Co-constructed Learning with Four MPD Elements	
	MTE	MGA-Designer
Entry	➢ Exploring what is the "ground" and when and how to build it. [goals]	➢ This concept is one of the statistical concepts included in standards and textbooks but hasn't been designed yet. [goals]
Attack	➢ Employing proper learning theories in designing MGAs. [theories] ➢ Furnishing students with the two factors (i.e. statistical concepts) for decision-making	➢ Insufficient content knowledge (i.e. statistical concepts) leads to an incomplete design of this module. [theories] ➢ Rules for playing the games in this module are too complicated. Students

	while playing the games. [theories/contexts]	were not able to acquire and use the concept while learning. [context] ➢ Lacking a complete pilot experiment of the module. [structure/contexts]
Review	➢ Following the three phases (entry, attack, review) to structure the MGA module. [structure]	➢ A deficient structure with less opportunity was provided for students to "review/reflect" in this module. [structure]

- Case II—Entry

In another case, MGA-designer A initiated the design of a new MGA module "Traffic Lights" after he experienced difficulties in designing the previous module. In the fifth class, he proposed this newly designed module, which was a comparatively successful case. Regarding to the goal of this module, it is expected that students are able to inspect the "possible position" of the traffic lights, be conscious of the "possible position" for judging what "the correct figure" is, and deduce the "consequences" based on the "causes/prerequisites". Experiencing and practicing "logical reasoning" is the core concept of this module [goals]. Three activities are included in this module: preparatory activity, formal activity, and PK activity [structure]. Through the preparatory activity, students can be practically familiar with the "figures of the traffic lights at the intersection". In this stage, the MGA-teacher shows the figure of the traffic lights to all students, and then ask students "where have you seen such figure of the traffic lights before".

In the formal activity, the MGA-teacher exhibits "a figure of the traffic lights" and asks all students to conjecture "which position of the lamp is failure". Also, if the lights are all normal, what is the correct figure of the traffic lights? Students will work in groups to discuss possible answers and present to the whole class. In addition, three consecutive figures of the traffic lights are given for further discussions. The last stage is the PK activity, where every team (2 students per team) has to design "two continuous 'countdown signals' (i.e. showing how many seconds left)" of the traffic lights (like ); however, at least one signal (seconds/figures) is not working well. Let other teams write down possibly correct figures of the traffic lights. If the answer is correct, the team gets 1 point; there are 10 rounds for each team, and then count the total score of each team. If needed, adjusting the difficulty level according to students' actual performance.

After his presentation, MTE1 showed his endorsement of this module [theories/contexts]:

"The essence of this logical reasoning activity is similar to the "three-view drawing", which is a determinative factor of a solid shape. Through providing three consecutive figures of the traffic light where there is a prerequisite condition of "one second" difference between two consecutive figures, this "simple and single" condition gives students a decisive state for logical thinking and reasoning. This is a good design!" (OB-102816)

MTE2 also mentioned that, "this module is a "real-life related" design, which is drawn from students'/everyone's life experience" (OB-102816) [contexts]. "We can see traffic lights with the countdown seconds and a green walking guy at almost every intersection. This is also the very first lesson that is taught from the kindergarten level" (OB-102816), said MGA-designer B. In fact, because of having a successful design of creative learning activities before in which he merged a real-life situation into a mathematical concept, he thought it might be a good idea that using the "traffic light" context to teach students "logical reasoning—"if…then…"(IN-102816) [goals].

Besides, seeing "broken" traffic lights on the street are a common phenomenon in our daily lives.

However, MGA-designer C raised a question about the formal activity that she thought giving "three consecutive figures" after recognizing "one figure" were complicated; "two consecutive figures" would be better…which seemed to be not that difficult, compared to three figures" (OB-102816) [contexts]. Moreover, MGA-designer E referred his concern to the preparatory activity, where "seeing" is the only way to figure out how this traffic light (i.e. countdown seconds) worked, by reflecting back to the main theoretical framework of "doing mathematics in the real world" [theories]. These concerns and discussions did help MGA-designer A to think about possible ways to improve the design of this module.

- Case II—Attack

Since there is no directly correspondent unit/content in current textbooks that teaches "logical reasoning", MGA-designer A decided to conduct his experimental teaching in four grade levels, i.e. 3rd to 6th grade [structure]. Based on those concerns and discussions raised in the previous class, he slightly adjusted the first draft of this module and tried to find out how it worked or not. First, in the preparatory activity, he asked students (in pairs; in every grade level) to write down the "units digit" number (0 to 9) on the learning sheets, following a writing practice of the "two digits" light (i.e. units and tens digits) [contexts]. In this way, students had more opportunities to "physically experience" how these lights worked by writing them instead of just "seeing" them (OB-120216) [goals/theories].

With regard to the formal activity, since MGA-designer C raised the question of complexity, MGA-designer A revised this part to include four sub-activities in order to guide students' learning in a sequential order, which was beneficial for students' concept development (IN-120216) [structure]. The revised version led students to practice the conjecture of "which position of the lamp is failure" by the order of "only unit digit (i.e. only 'ones', 0~9)" to "two digits (00~99, but only the 'units digit' is failure)" [sub-activity 1 & 2], two consecutive "countdown" numbers [sub-activity 3], and three consecutive "countdown" numbers [sub-activity 4]. For the sub-activity 2, students were asked to practice to conjecture the possible failure(s) of the "two digits" light. Different from the sub-activity 1 (only unit digit), one more digit was "actually adding more challenges for these students; especially younger students (e.g. 3rd and 4th grades), they kept discussing possible answers but it looked like they got stuck…", said MGA-designer A (OB-120216). He reflected on this struggling moment: "I thought I didn't give them enough time to think and discuss" (OB-120216) [contexts]. MTE2 addressed his concern about this issue: "since you added one more digit in this sub-activity, there would be three possible "failure" combinations of "only unit digit", "only tens digit", and "both digits" (OB-120216). He thought this complexity might be the reason that younger students got stuck in this sub-activity.

With regard to the sub-activity 3, MGA-designer A explained that, "compared to the sub-activity 1 & 2 that students needed to make judgments based on the given "failure" appearance of the light (one or two digits), they had to conjecture possible answers according to the context of the two consecutive "countdown" numbers" (OB-120216) [contexts]. However, MGA-designer C raised the same concern of complexity that was similar to the aforementioned struggling:

MGA-designer C: Did you try to control the failure situation to "only unit digit?

MGA-designer A: Actually, I didn't mention that.

MGA-designer C: Then, it would be a bit complicated, as you mentioned above.

MTE2: What was your objective of this activity? You had to carefully think about it [goals].

MGA-designer A: Right! I just thought I wanted them to practice in various situations…so that I didn't control the failure context.

MTE2: I thought we all agreed that learning the logical reasoning "if…then…" was the main goal of this activity [goals]. However, too often, we didn't set up the causes/prerequisites properly. Here you might want to give a limited cause/prerequisite at the beginning stage of this sub-activity that "if only the unit digit was broken", which would be better for younger students to conjecture the possible answers.

MGA-designer C: I thought this suggestion was good. In this way, students could truly learn
in a sequential order (easy to difficult), which was also echoed to the struggling issue [structure]. (OB-120216)

This discussion indeed helped MGA-designer A to revise this module to the final version at the last (8th) PD class, which had more acceptable sequences and controls of all given conditions.

Regarding the PK activity, students' cognitive development did have an influence on their designs. For instance, MGA-designer A recalled his pilot study result that only some of 6th graders were able to design more difficult situations (PK questions), in which the two digits were both broken (OB-120216). In addition, PK questions designed by lower grade levels could be easily solved by high graders; on the contrary, PK questions designed by higher grade levels would be too difficult to be solved by lower graders. MTE2 reminded him that:

"Like penetrating a magic show, one had to understand the theory or technique(s) behind the show as well as requiring a lot of practices. Back to the design of the PK question or your instruction design, either students or teachers needed to integrate what they learned before in order to produce more complicated/difficult questions or instructional plans." (OB-120216)

These two findings and MTE's words reminded MGA-designer A about the issue of complexity, which he, later on, thought that "5th graders might be the best fit of this module" (IN-120216). However, MGA-designer A kept his design that this module would be still appropriate for students from the 2nd grade level, which was noted on the final version of this module (RR-120916). Besides, he added a reflection section on his learning sheets that included two questions, where students need to work individually after finishing all activities (DO-120916): (1) Giving a "two digits" traffic light that is not fully functioning, which position of the lamp is failure? If the lights are all normal, what is the correct figure of the traffic lights? Please illustrate your idea and how you figure out the answer(s). (2) Under what kind(s) of the situation, will it be easier for you to find out the failure lamp(s)? Similarly, what situation is more difficult to find out? Through this reflection activity, "it is to assist students to deeply re-think about the whole process of this module, in which their understanding will be reflectively promoted" (IN-120916).

- Case II—Review

According to the actual conditions he confronted in all experiments (i.e. four different grade levels) and those suggestions that were raised by MTE2 and MGA-designer C during his presentation in the 7th class, MGA-designer A reflected carefully upon the theoretical essence of "three-view drawing" addressed by MTE1 [theories/contexts], as well as the main theoretical framework of "doing mathematics in the real world" remarked by MGA-designer E [theories]. He, in consequence, added a series of sequential practicing activities in the learning sheets, which are able to furnish students with abundant opportunities to scaffold their understandings of using the logical reasoning "if…then…" to conjecture possible answers in all activities of this "traffic light" module [structure]. For instance, three parts were added (see attachments for details): (1) physically practicing how to write the traffic lights (from one digit to two digits) in the preparatory activity—with 1st part of the learning sheets; (2) sequentially learning through four sub-activities of the formal activity (from one two-digit "countdown" seconds to two consecutive "countdown" seconds)—with 2nd part of the learning sheets; (3) adding extra cards (i.e. card set A for one two-digit figure with failure lamp(s); card set B with two consecutive countdown figures with failure lamps)—in the last part of the learning sheets (DO-120916). By using these practicing activities in the learning sheets (both writing and card sets), students are able to visualize how these traffic lights work, i.e. either working well or with failure lamps. These learning procedures are truly beneficial for not only cultivating students' mindsets (i.e. both actually doing math and doing math in their minds), which is similar to the essence of employing "three view drawing" for establishing the capability of "visualization", through plenty of physically exercises [theories/contexts] but also scaffolding the development of their capability of the logical reasoning "if…then…" [structure] (RR-120916). Besides, as MGA-designer A claimed in the last interview, "those card sets retain the flexibility of providing future learning opportunities for certain students with better performances" (IN-120916), such as for students at higher grade levels, where they may produce/design more complicated/difficult PK questions later.

Table 2. Successful case—"traffic lights"

Problem-Solving Phases	Parallel Co-constructed Learning with Four MPD Elements	
	MTE	MGA-Designer
Entry	➢ Using "simple and single" condition to give students a decisive state for logical thinking and reasoning. [goals] ➢ Employing a proper theory to design this module. [theories] ➢ A "real-life related" design that is drawn from students'/everyone's life experience. [contexts]	➢ Based on previous experiences of successful designing creative learning activities, a real-life situation ("traffic light" context) was merged into a mathematical concept (logical reasoning—if…then…). [goals/contexts] ➢ Applying the main theoretical framework of "doing mathematics in the real world" in the design. [theories] ➢ A struggling issue was raised about the complexity of all activities. [contexts/structure]
Attack	➢ Similar struggling issue of the complexity was addressed in the sub-activities of the formal activity—which and/or how many digits with failure lamps, which led younger students to get stuck. [contexts] ➢ Reflecting back to the main goal of this activity and reminding him to (1) give a limited cause/ prerequisite at the beginning stage of the formal activity and	➢ Writing practices were added for giving more opportunities to "physically experience" how these lights worked. [goals/theories/contexts] ➢ Four sub-activities in a sequential order for assisting students' concept development. [structure] ➢ Conducting pilot experiments to student in four grade levels [structure/contexts] ➢ Not fully control the failure situation within the practicing activities to "only units digit, which again caused struggling issues when students tried to conjecture possible

	(2) be aware of the complexity issue. [goals/contexts] ➢ Furnishing. [theories/context]	answers. [contexts] ➢ Reflecting to the pilot experiments for later improvements: (1) not offering enough time for students to think and act; (2) the struggling issue related to the complexity and the theory of "doing math in the real world". [structure/theories/contexts]
Review	➢ Following the theoretical essence of "three-view drawing" addressed by MTE1 and the main theoretical framework of "doing mathematics in the real world". [theories/contexts]	➢ Adding both (1) a series of sequential practicing activities in the learning sheets for the scaffolding purpose and (2) a series of open-ended questions for students to reflect upon their own learning processes. [structure/theories/contexts]

Discussion and Implication

According to the findings of this study, MGA-designer A failed to complete the design of the module "throwing darts", while the second module "traffic lights" was successfully designed. Grounded on the MPD model with the four elements (i.e. goals, contexts, theories, and structure) proposed by Sztajn, Campbell, and Yoon (2011), Lin, Yang, and Wang (2016) claim that the four-element model is employed in the JDM project for conceptualizing the professional development programs, where the goal is to help students engage in mathematics learning cognitively and affectively. The three-phase (i.e. entry, attack, and review) "mathematical problem solving" model, proposed by Mason, Burton, and Stacey (1982), is also employed in the MGA-designers' PD process of JDM project, which furnishes MGA-designers with abundant personal experiences of thinking and doing mathematics in the real world, solving real-life problems, and working collaboratively in this design-based PD process (Lin, Yang, & Wang, 2016). As shown in table 1 and 2, with his first failure experience of the module "throwing darts", MGA-designer A clearly sets up his goal of the module "traffic lights", employs appropriate theories while designing, creating real-life situated learning contexts and a series of sequential activities to structure the whole learning process. In addition, a complete "problem-solving" process is observed, where a reasonable entry phase is initiated based on previous knowledge and experiences, a full attack is accomplished with pilot experiments in four different grade levels, and a reflective review is performed during the whole designing process. Therefore, MGA-designer A is able to finally achieve his original goal—merging a real-life situation (traffic lights) into teaching students a mathematical concept (logical reasoning "if…then…"). On the contrary, his first module design is conducted based on an unclear goal, problematic theory use with insufficient content knowledge, defective contexts with complicated playing rules, an incomplete pilot experiment, and a deficient structure with less opportunity for him and his students to review/reflect. Compared to this successful case, these defective elements comprise incomplete problem-solving phases in his designing process. As a result, based on MGA-designer A's learning process within the PD program, it shows that the use of both the four-element MPD model and the three-phase problem-solving model provides essential supports in scaffolding MGA-designers' professional development, where they can not only engage in the learning process cognitively and affectively (like students) but also implement how to design personally and simultaneously (as designers). Consequently, this PD design is beneficial for MGA-designers' professional development.

As mentioned in the theoretical framework section, the PD process in the JDM project employs the co-constructed learning approach (Chen, Lin, & Yang, 2018) and the dynamic nature (Javaski, 2008; Zaslavsky, 2008) in the task design and implementation process, where MTEs and MGA-designers can learn collaboratively in this co-constructed structure. Within this co-constructed learning structure, MTEs and MGA-designers share symmetrical roles in the PD program: MTEs are responsible for designing PD workshops, guiding and educating MGA-designers, and reflectively monitoring/evaluating MGA-designers' learning process. Multiple roles of MGA-designers include: learners within the PD workshops, designers and pilot experimenters of the MGA modules, and monitors that reflectively assess students' learning process. Grounded on this analogous essence of multiple roles, MTEs and MGA-designers authentically and collaboratively learn both from each other and within the PD processes. Especially for MGA-designers, they practically learn how to design MGA modules and simultaneously execute the designing tasks within these collaboratively co-constructed PD activities, which echoed Chen, Lin, and Yang's (2018) viewpoint that students, teachers, and teacher educators are all learners within the PD program and its implementation process. In fact, since they are both learners and designers/executers, the whole MGA-designers' PD process, composed of the four elements, leads all MGA-designers to collaboratively establish their own learning process in this co-constructed framework. In this study, through presenting one MGA-designer's two task-design processes, starting from the first failure case to the second successful case, the designated MGA-designer's learning process of designing MGA modules are thoroughly exhibited as empirical evidences of the effectiveness of the JDM MGA-designers' PD program.

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