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Didactic Scenarios for Teaching the Concept of Quadratic Function and its Applications in the Framework of Physics Teaching

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Abstract

This work develops a didactic sequence whose central axis is the modeling of physical phenomena, which leads to a quadratic model through direct experimentation or the use of simulations using the analysis of variables, hypothesis formulation and their verification. The objective is to present the modeling process through the concept of a quadratic function in the solution of physical phenomena, in a group of ninth-grade students, in a Public Educational Institution of the Department of Norte de Santander, Colombia. The methodology followed was qualitative, descriptive and interpretative in a case study that adopted the phenomenological design. It is concluded that the didactic sequence uses examples and applications of mathematics in areas such as physics, which facilitates the understanding of the concepts, in this case, the quadratic function.

Keywords: Mathematical Modeling, Quadratic Function, Physical Phenomena, Model, Experimentation.

1. INTRODUCTION

The concept of function is important in teaching mathematics as it is part of models of both natural and social phenomena. Thus, using functions, it is possible to describe the behavior of natural events of a physical (description of movements), chemical (radioactive disintegration) and biological (population growth) nature. It is also possible to detail social situations, such as demographic and economic aspects among others, in order to model them and find representations that allow predicting behavior in a general way (Ballester, 2009; González, 2016; Arrarás, & Cappello, 2019).

In elementary and middle school education, in the Colombian educational context, the concept of function contributes to the structuring of variational thinking and analytical systems, as established in the curricular guidelines and standards (Mineducación, 1998; 2006), since it promotes in the student competences and skills for observation, measurement, data recording, making different graphical representations, which allow the identification of variables and the establishment of functional relationships applicable to real contexts. Several researches in the field of mathematics education have studied the concept of function from the historical (Sánchez & Valcárcel, 1993), epistemological (Ruiz, 1998; Sastre et al., 2009) and didactic (Villa & Ruiz, 2011) points of view.

Taking into account this background and the curricular guidelines of the Colombian Ministry of National Education about mathematical modeling as a general process for the acquisition of mathematical knowledge, this work aims to describe through the design of a didactic sequence the use of the concept of function as an application in the modeling of physical situations leading to quadratic models, to facilitate student learning.

1.1 Teaching the concept of function

In Colombia, the teaching of the concept of function, as any other mathematical concept, has gone through the so-called modern mathematics that intended to bring students closer to the study of pure mathematics that approached abstraction but moved away from the development of competencies and visualization skills interpretation and modeling, which caused some negative impact on students and in general on the teaching learning of mathematics (Gómez, 2021; Arboleda, 2019). After this, a school reform was initiated that sought to overcome the difficulties presented; the systems approach is oriented towards conceptualization and the development of competencies and thinking by students, as well as the work of mathematics to its different branches (Vasco, 1985). Thus, the approach proposes three aspects to organize the curriculum: general processes related to learning, basic knowledge related to the specific processes and systems of mathematics, and the context related to the environment surrounding the student (Mineducación, 1998), the latter being the one that allows bringing students closer to mathematics, according to the student's context and allowing them to improve their ability to solve problems close to reality, understand the importance of mathematics, and its usefulness in everyday life (Bosch et al., 2006). Therefore, the approach is to bring students closer to everyday mathematics, where they can see applications, its usefulness and relationship with other sciences, but which involves processes in which they can move from the concrete to the abstract.

1.2 Modeling and quadratic function

According to the mathematics curricular guidelines (Mineducación, 2006), modeling, a contraction of the terms modeling and education, i.e., modeling = Modeling + Education (Córdoba-Gómez et al., 2022), is one of the five general processes present in mathematical activity, related to problem-solving, and is a tool for the construction of mathematical concepts. In mathematical modeling, the starting point is a real situation that must be modeled according to the conditions of the problem and thus brought to the classroom (Muro et al., 2007).

The quadratic function model explains various situations in real life (Cetina et al., 2016); some of them can be found in economics when optimizing revenue, cost and utility functions (El Alabi & Milanesi, 2016), in geometry when working with container filling, (Tinoco, 2020), variability of the dimensions of a figure and from the physical point of view, when working problems related to kinematics such as uniformly accelerated motion, free fall, projectile launching, (Ramirez, 2015).

To transfer the real situation to a mathematically posed problem (Martínez et al., 2015), the following activities are proposed: identify mathematics in a general context, schematize, formulate and visualize a problem in different ways, discover relationships and regularities, recognize isomorphic aspects in different problems, transfer a real-life problem to a mathematical and known problem (Camelo et al., 2016).

When the real-life problem has been transferred to a mathematical problem by which acquired knowledge is used to discover regularities, relationships and unknown structures (De Lange, 1987) it can be represented with a formula, show regularities, generalize and validate the model (Montero & Badía, 2011). In the classroom, the modeling process is considered a scientific activity that involves establishing relationships between the real world and the mathematical world.

2. METHODS

2.1 Design

The methodology was qualitative, descriptive and interpretative in a case study that adopted a phenomenological design, to explore, describe and understand the students' experiences that arose from a modeling process in the school environment. The information was obtained from the students' productions, short video and audio recordings of some of the discussions that arose during the teamwork.

2.2 Scenario and actors

The study context was a ninth-grade mathematics class of a public educational institution in the Department of Norte de Santander, Colombia (17 boys and 11 girls), voluntarily divided into teams of 4 students. The selection of the working group was not random but was made by convenience. The experimentation was carried out in the classroom and its surroundings during two-hour sessions, after explaining the concept of function and quadratic function.

2.3 Materials and staging of activities

A 6-question diagnostic test was applied to diagnose difficulties and identify preconceptions about quadratic functions.

The didactic sequence was designed and implemented, which has as its central axis the modeling from problem situations of physical contexts (Briceño & Buendía, 2016) and was developed according to the curricular standards of mathematics and natural sciences (Mineducación, 2006) in aspects related to variational thinking such as: describing situations, finding regularities, discovering patterns, aspects that allow approaching scientific knowledge naturally, formulating and answering questions, identifying and modifying variables related to natural phenomena. It was based on constructivist principles to achieve meaningful learning and sought to strengthen the teaching-learning process of the concept of quadratic function, taking into account the following stages: description of the problem situation, hypothesis formulation and identification of variable magnitudes, real or virtual experimentation, data collection and analysis, formulation, generalization and identification of the mathematical model and finally the validation of hypotheses through the use of the mathematical model.

Finally, a post-test was applied to determine the comprehension levels achieved by the students after having developed the didactic sequence.

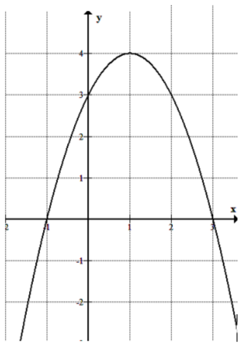
3. RESULTS AND DISCUSSION

3.1 Diagnostic analysis

The diagnostic questionnaire was designed to be applied to students before the implementation of the didactic sequence and its purpose is to know what students know about quadratic functions.

Table 1 Diagnostic question and some answers

Ask	**Difficulties encountered**
Define in your own words what a quadratic function is.	Most of the students did not clearly know what they were doing of the definition of the quadratic function.
Complete the table of values for the function: y= f(x) = x^2 -3x-1 showing the step-by-step procedures and operations to obtain the values of y or f(x).	When performing basic operations such as adding integers, they applied the law of multiplication signs; when squaring a negative quantity they obtained negative values, when multiplying by zero they obtained the same factor as a result, when solving the product of 2 factors they inadequately applied the law

	of signs, i.e. when multiplying 2 integers of different signs they obtained a positive result or when multiplying two negatives the result was negative.
According to the following graph, please answer:  a) The graph intercepts the X-axis at the points: b) The point of intersection with the Y axis is: c) The vertex is the point: d) The line of symmetry is: X =	Most students did not get the question right because they were unclear about the elements of a quadratic function. It was also evident that the students did not consider a point as a coordinate of the form p(x, y) but as a single component of x or y in isolation.
Draw in the Cartesian plane the graph of a parabola taking into account the table of values x | f(x) -2 | 5 -1 | 0 0 | -3 1 | -4 | -3 | 0 | 5	For the students this question was the one with the correct answers. Still, there were situations where students had difficulties locating points on the Cartesian plane, since they only located the respective components on each axes and then joined them with straight lines. Others invert the coordinates of x by y and vice versa.

x	f(x)
-2	5
-1	0
0	-3
1	-4
	-3
	0
	5

The construction of a suspension bridge that consists of two 20 m high towers separated at a distance of 50 m, in addition it is desired to place vertical cable or suspenders every 5 m and the parabolic cable that supports the bridge touches the ground in its central part, what length should have each vertical cable and how much vertical cable is required in total.	In this question it was evident that the students could not solve the problem. The closest was that some had some schema but could not make a connection with the given situation.
Mention 2 applications of quadratic functions in everyday life.	This question corroborated that students do not know the applications of quadratic functions.

According to the analysis of the diagnosis, some difficulties of the students are observed, such as locating points in the Cartesian plane, identifying the elements of a quadratic function, and solving problems that use quadratic functions, among others. The didactic sequence is formulated based on the results of the diagnosis.

3.2 Teaching sequence

The didactic sequence proposed has to do with the implementation of quadratic models from physics whose objectives are to identify and understand a function from different representations; identify and classify variables involved; formulate different hypotheses; analyze the behavior of the variables involved in the situation; build the mathematical model that meets the conditions of the physical situation and graph the situation through the hypotheses made.

As preconceptions from the point of view of mathematics, students must have clarity about ordered pairs, Cartesian products, relations, function, interpretation and identification in different representations, domains, ranges, dependent variables, independent variables, evaluation of functions, maximum or minimum point, points of intersection with the axes, among others. From the point of view of physics, it is necessary to have clarity about the concepts of uniform motion, uniformly accelerated motion, free fall, parabolic and semiparabolic motion, among others. Therefore, the central themes on which the didactic sequence is focused are data by the concept of quadratic function, verbal representation, tables, graphs, evaluation, points of intersection with the axes, maximum or minimum point, the behavior of the parameters in the quadratic function and construction of mathematical models related to physical situations.

Finally, the didactic sequence in teaching activities allow experimentation and simulation with software such as Geogebra, Modellus or Excel of situations by solving problems related to the context of physics such as free fall of a body, uniformly accelerated motion, projectile launching, among others, so that students observe the use of quadratic functions as tools that explain real-life situations.

3.3 Analysis of final evaluation and comparison with a diagnosis

Next, we proceed to the analysis to determine how the didactic sequence impacted the students' appropriation of quadratic functions. In this section, an analysis is made between the pretest and the posttest, that is, before and after the didactic sequence, for which a parallel was made between the correct procedures in both cases.

Table 2 Comparison of pretest vs. posttest results.

			Pretest	Posttest
Question 1	Successful answer		7%	64%
	Moderately accurate answer		57%	14%
	Did not resolve		36%	21%
Question 2	Complete the table of values correctly		14%	61%
	Shows difficulties in completing the table of values.		75%	25%
	Did not resolve		11%	14%
Question 3	Identifies the intercepts with the x-axis.	Correct	0%	68%
		Incomplete	14%	21%
		Does not resolve	86%	11%
	Identifies the intercept with the y-axis	Correct	4%	75%
		Incomplete	75%	14%
		Does not resolve	21%	11%
	Identifies the vertex	Correct	21%	78%
		Incomplete	57%	14%
		Does not resolve	21%	7%
	Identifies the line of symmetry	Correct	4%	68%
		Incomplete	46%	29%
		Does not resolve	50%	4%
Question 4	Locate all points correctly in the plane and plot their respective graphs.		50%	86%
	Does not locate all points correctly		36%	10%
	Does not resolve		14%	4%
Question 5	Draw a picture showing the situation	Correct	29%	79%
		Incomplete	18%	18%
		Does not resolve	54%	4%
	Pose the equations of the situation	Correct	0%	71%
		Incomplete	14%	18%
		Does not resolve	86%	11%
	Solve the problem using the quadratic function	Correct	0%	68%
		Incomplete	11%	25%
		Does not resolve	89%	7%
Question 6	Yes		14%	68%
	No		14%	21%
	No response		71%	11%

According to Table 2, in question 1 it is observed that students significantly improved their performance against the definition of the quadratic function. With respect to question 2, students also improved their performance because they made fewer arithmetic errors to complete the table. For question 3 students had to identify the coordinates of the elements of the graph of the quadratic function. Comparing the final evaluation results, it is observed that students significantly improved their performance in this question. In question 4, students also improved their performance by moving from the tabular representation to the graph of the function since they improved in the location of points in the Cartesian plane. For question 5, students had to solve a problem situation that follows certain steps: draw or make a scheme that represents the situation posed, pose the equations that describe or model the problem and make use of the quadratic function, in the resolution and analysis of the problem, in all these aspects an improvement of the students in the final evaluation is evidenced. Finally, for question 6, students have also improved in identifying and understanding the applications of quadratic functions. In summary, it can be affirmed that the didactic sequence implemented positively affected problem-solving.

5. DISCUSSION

The didactic sequence suggested to teach the quadratic function is focused on the process of mathematical modeling from problem situations in physical contexts; it approaches the student to scientific knowledge in a natural way and thus develop their competencies, as evidenced by the works of Briceño & Buendía (2015; 2016). In addition, they favor the significant understanding of the concept of quadratic function, its behavior, the construction of its graphical representation, the description and modeling (Rey et al., 2008; Chávez et al., 2018) of some applications coming from physics.

The modeling process is achieved from some applications related to physics phenomena that lead to quadratic-type functions and facilitate the student's conceptualization through: the identification of variables and their relationships, graphical and algebraic representation of the relationship obtained, as well as the use of this to predict future behaviors of the variables that affect the phenomenon under study (Cetina et al., 2016; Recio-Avilés, 2018).

The use of laboratory practices and/or suggested simulations facilitates the student's construction and assimilation of the concepts, and the experimentation of properties, particularly the variation of the parameters of a quadratic function.

5. CONCLUSIONS

Regarding the evaluation of the didactic sequence, it is evident that the students' results improved between the diagnostic and the final test, so it is concluded that its implementation was feasible because the students improved their competencies regarding quadratic functions, which led students to improve their variational thinking. It is important to express that the present study did not have as a priority to measure the dimensions and levels of comprehension reached by the students but to describe the implementation of the didactic sequence under the approach used for its design so future studies that can determine these indicators are recommended.

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